Multiple Choices:

1. Answer: A or D.

The phone is moving at constant velocity. The only force acting on the phone is the earth gravity.

Remark: The answer is D. However, since the question asks for "instantaneous scalar readings", it is not unreasonable to also consider A as a valid answer.

2. Answer: D.

Stage 1: h1 = (1/2) (2.5 g) $(5)^2$ = (31.25 g) m

Stage 2: v = (2.5 g) (5) = 12.5 g ms^{-1}; h2 = $(12.5 g)^2$ / 2g = (78.125 g) m

Maximum Height = (31.25 + 78.125) g = (109.375 g) m

3. Answer: C.

a = v^2 / r West = 5^2 / 5 West = 5 ms^{-2} West

4. Answer: B.

F = (0.01 x 200 x 600) / 60 = 20 N.

5. Answer D.

Apply Newton's law of universal gravitation at the Earth's near side and its far side.

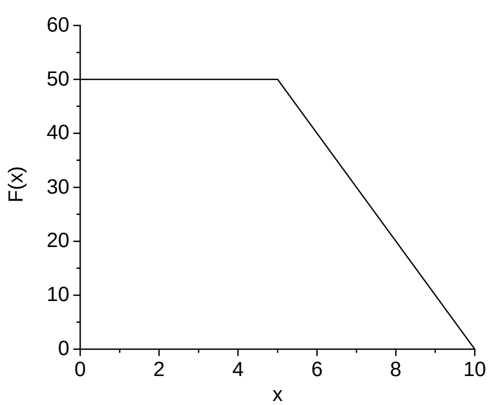
6. Answer B.

Bottom Wire: $T_2 = \frac{m}{2\cos\theta}\left(\frac{v^2}{r} - g\cot\theta\right) = 28.43\ \mathrm{N}$

Bottom Wire: $T_1 = T_2 + \frac{mg}{\sin\theta} = 38.23\ \mathrm{N}$

7. Answer E.

Work Done = Area under the following curve.



8. Answer E.

Apply $T = 2\pi\sqrt{\frac{m}{k}}$ to perform calculation.

9. Answer D.

Two normal modes.

First Mode: $f_1 = \sqrt{\dfrac{g}{l}}$

Second Mode: Consider the center of mass of the 2 masses being unchanged, the effective spring constant is $K_1 = \dfrac{2m}{m}K = 2K$

$$\Rightarrow f = \sqrt{\frac{g}{l} + \frac{K_1}{m}} = \sqrt{\frac{g}{l} + \frac{2K}{m}}$$

10. Answer A.

$$\frac{GMm}{r^2} > m\omega^2 r \qquad \Rightarrow \qquad T > \sqrt{\frac{3\pi}{\rho G}}$$

11. Answer C.

$$\frac{\rho_{liquid}}{\rho_{water}} = \frac{F_{liquid}}{F_{liquid}} = 2$$

12. Answer D.

Energy Conservation:

$$\frac{1}{2}(m+M)v^2 = (m+M)gh$$
$$v = 0.313\,\mathrm{ms^{-1}}$$

Momentum Conservation:

$$mv_b = (m+M)v$$
$$v_b = 62.9\,\mathrm{ms^{-1}}$$

Initial Kinetic Energy:

$$KE = \frac{1}{2}m{v_b}^2 = 19.8\,\mathrm{J}$$

13. Answer B or C.

$$x = v_0 t \cos\theta$$

$$y = v_0 t \sin\theta - \frac{1}{2} g t^2$$

When the ball hits the inclined plane again,

$$y = -x \tan\theta$$

Eliminating *x*,

$$-v_0 t \sin\theta = v_0 t \sin\theta - \frac{1}{2} g t^2$$

$$t = \frac{4 v_o \sin\theta}{g}$$

$$x = v_o (\cos\theta)(t) = \frac{4 v_o^{\ 2} \sin\theta \cos\theta}{g}$$

$$s = \frac{x}{\cos\theta} = \frac{4 v_o^{\ 2} \sin\theta}{g}$$

Remark: Students reading the English version may choose answer B. Students reading the Chinese version may choose answer C. Due to this discrepancy, both answers are accepted.

14. Answer A.

15. Answer E.

Consider the system with 3 spheres:
a. sphere of density ρ and radius R with $y_1 = 0$ (center of mass).
b. sphere of density -ρ and radius R/2 with $y_2 = R/2$.
c. sphere of density 5ρ and radius R/2 with $y_3 = R/2$.
(b) and (c) are equivalent to a sphere of density 4ρ and radius R/2 with $y_4 = R/2$.

Center of mass of the new sphere:

$$Y = \frac{\sum m_i y_i}{\sum m_i} = \frac{\rho\left(\frac{4}{3}\pi R^3\right)(0) + 4\rho\left[\frac{4}{3}\pi (R/2)^3\right](R/2)}{\rho\left(\frac{4}{3}\pi R^3\right) + 4\rho\left[\frac{4}{3}\pi (R/2)^3\right]}$$

$$= \frac{R}{6}$$

16. Answer: C.
Potential Energy:

$$E = g\rho A\int z\,dz$$
$$= \frac{1}{2}g\rho A h^2$$

$$\frac{E}{A} = \frac{1}{2}g\rho h^2$$
$$= 5007.8h^2 \quad \text{[J]}$$
$$= 1.39h^2 \quad \text{[Wh]}$$

17. Answer D.

$$T = 2\pi\sqrt{\frac{l}{g}} \quad \Rightarrow \quad T_M = \sqrt{\frac{g_E}{g_M}}\cdot T_E = \sqrt{6}\cdot T_E$$

18. Answer B.

Using the equation $\frac{GM_{\text{Earth}}m}{R^2} = \frac{mv^2}{R}$ and period of the Earth T = 86400 s to calculate the height of a geostationary satellite.

19. Answer A.

For large angle oscillation, the equation of motion becomes nonlinear, and the approximation $\sin\theta \approx \theta$ is now no longer hold. The restoring force is proportional to $\sin\theta$, rather than θ; therefore its magnitude is less than in the case of SHM. A weaker restoring force also results in slower oscillation, that is, the period becomes longer.

20. Answer C.

It is a plot of $g = GM/R^2$, that is, g against (M/R^2).

Open-ended Questions

1. Venus Transit

(a) Kepler Law: $\frac{P^2}{a^3} = $ Constant, P = Orbit Period, a = Orbit Radius

$$\left(\frac{a_{Earth}}{a_{Venus}}\right)^3 = \left(\frac{365}{225}\right)^2$$

$$\frac{a_{Earth}}{a_{Venus}} = 1.3806$$

(b) $$\frac{A'B'}{AB} = \frac{A'V}{AV} = \frac{A'V}{AA' - A'V} = \frac{1}{a_{Earth}/a_{Venus} - 1} = 2.6273$$

$$A'B' = 2.6273 \times 1800 \text{ km}$$
$$= 4729 \text{ km}$$

(c) Diameter of the Sun = 4729 km x 290 = 1.37x10^6 km

(d) Let v_E = velocity of Earth relative to Sun

Since planetary velocity is given by $v = \sqrt{\frac{GM_{\text{Sun}}}{r}}$, velocity of Venus relative to Sun = $v_E \sqrt{\frac{a_E}{a_V}} = 1.1750 v_E$.

As observed from Earth, velocity of Venus = $v_E\left(\sqrt{\frac{a_E}{a_V}} - 1\right) = 0.1750 v_E$, and velocity of Sun = -v_E.

Projected on to the surface of Sun, velocity of the shadow of Venus = $v_E\left(\sqrt{\frac{a_E}{a_V}} - 1\right)\frac{a_E}{a_V} = 0.2416 v_E$.

Hence the velocity of the shadow of Venus sweeping on the surface of Sun =

$$v_E\left[\left(\sqrt{\frac{a_E}{a_V}} - 1\right)\frac{a_E}{a_V} + 1\right] = 1.2416 v_E$$

$$v_E = \frac{2\pi r_E}{T_E} = \frac{(2\pi)(1.5 \times 10^{11})}{(365)(24)(60)(60)} = 29886 \text{ m/s} \quad \text{or}$$

$$v_E = \sqrt{\frac{GM_{Sun}}{r_E}} = \sqrt{\frac{(6.67 \times 10^{-11})(1.99 \times 10^{30})}{1.5 \times 10^{11}}} = 29747 \text{ m/s}$$

Time difference = $\frac{4729}{(1.2416)(29886)} = 127 \text{ s} = 2.12 \text{ min}$ or $\frac{4729}{(1.2416)(29747)} = 128 \text{ s} = 2.13 \text{ min}$

2. Terminal Velocity of Free Falling Object

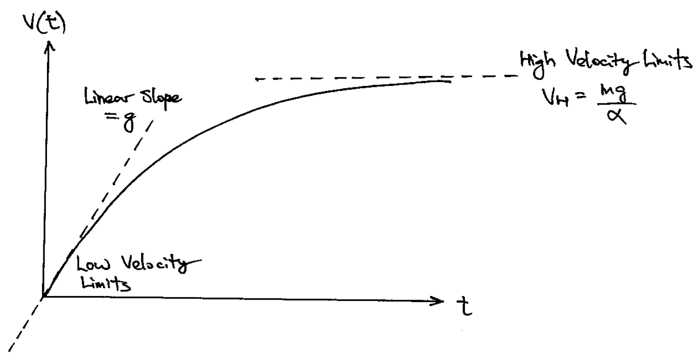
(a) $$m\frac{dv}{dt} = \underbrace{F_g}_{\text{Gravity}} + \underbrace{F_d}_{\text{Drag Force}} = mg - \alpha v$$

At low velocity limits, $F_d \to 0$.

$$m\frac{dv}{dt} \approx F_g \qquad \Rightarrow \qquad v_L \approx gt$$

At high velocity limits, $m\frac{dv}{dt} \to 0$.

$$F_g + F_d \approx mg - \alpha v \qquad \Rightarrow \qquad v_H \approx \frac{mg}{\alpha}$$

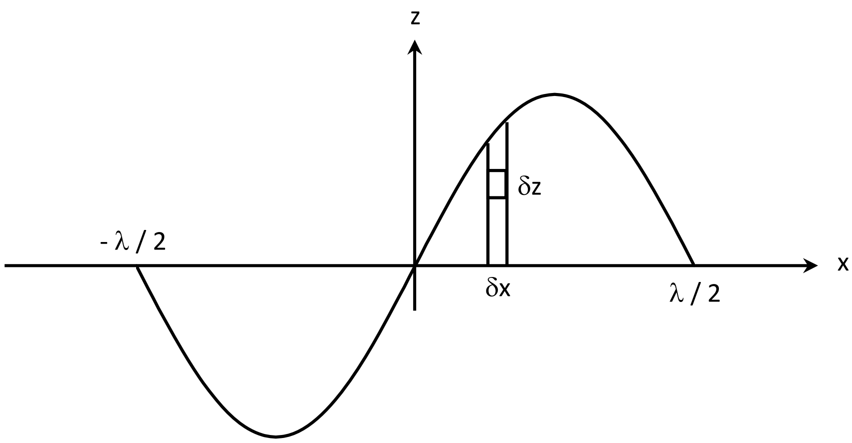


(b) $$mg = \frac{1}{2}C_d A\rho v^2$$

$$v = \sqrt{\frac{2mg}{C_d A\rho}} = \sqrt{\frac{2(2\times10^{-3})(9.8)}{0.5\times\pi\times(0.02)^2\times1.2}}$$
$$= 7.2\ \text{ms}^{-1}$$

(c) $$t \approx \frac{v}{g} = \frac{7.2}{9.8} = 0.7\ \text{s}$$

3. Total Energy in a Surface Wave



(a) Gain in potential energy δV from –z to z of an elemental mass δm = ρ δx δz:

$$\delta V = (\delta m)(g)(2z) = 2\rho g z(\delta x)(\delta z)$$

Total Potential Energy:

$$\begin{aligned} V &= 2\rho g \int_{x=0}^{x=\lambda/2} \int_{z=0}^{z=A\sin(2\pi x/\lambda)} z\,dz\,dx \\ &= 2\rho g \int_{x=0}^{x=\lambda/2} \left[\frac{z^2}{2}\right]_{z=A\sin(2\pi x/\lambda)} dx \\ &= \rho g \int_{x=0}^{x=\lambda/2} A^2 \sin^2(2\pi x/\lambda)\,dx \end{aligned}$$

Given that $\int \sin^2\left(\frac{2\pi x}{\lambda}\right) dx = \frac{x}{2} - \frac{\lambda \sin(4\pi x/\lambda)}{8\pi}$,

$$\begin{aligned} V &= \rho g A^2 \left[\frac{x}{2}\right]_0^{\lambda/2} \\ &= \frac{1}{4}\rho g A^2 \lambda \end{aligned}$$

Total PE per wavelength:

$$\frac{V}{\lambda} = \frac{1}{4}\rho g A^2$$

(b) Assume equipartition of energy (PE = KE), the total energy over a whole wavelength

$$E = \frac{1}{2}\rho g A^2$$

(c) Power of a Wave Period:

$$P_w = \left(\frac{1}{2}\rho g A^2\right) \times v_g$$

$$= \left(\frac{1}{2}\rho g A^2\right)\left(\frac{\lambda}{2T}\right) \qquad V_g = \frac{\lambda}{2T}$$

λ = Wavelength
T = Period

$$= \frac{\rho g^2 T A^2}{8\pi} \qquad \lambda = \frac{gT^2}{2\pi}$$

$$= \frac{\rho g^2 T H^2}{32\pi}$$

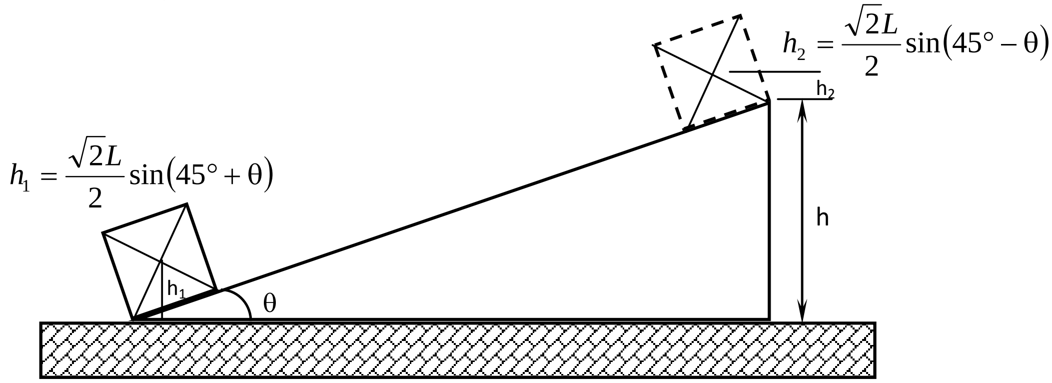
Wave amplitude is half of the wave height

4. A Sliding Block up a Slope Platform

(a) Momentum:

$$mv_o \cos\theta = (m+M)v_{Total} \qquad \Rightarrow \qquad v_{Total} = \frac{mv_o \cos\theta}{m+M}$$

Raise in Center of Mass, h'':



$$h'' = h + h_2 - h_1$$
$$= h + \frac{\sqrt{2}L}{2}\sin(45° - \theta) - \frac{\sqrt{2}L}{2}\sin(45° + \theta)$$

Applying the trigonometric identity: $\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$,

$$h'' = h - L\sin\theta$$

Energy:

$$\frac{1}{2}mv_o^{\ 2} = \frac{1}{2}(m+M)v_{Total}^{\ 2} + mgh''$$

$$\frac{1}{2}mv_o^{\ 2} = \frac{1}{2}(m+M)\left(\frac{mv_o \cos\theta}{m+M}\right)^2 + mgh''$$

$$\left(m - \frac{m^2\cos^2\theta}{m+M}\right)v_o^{\ 2} = 2mgh''$$

$$v_o^{\ 2} = \frac{2gh''(m+M)}{(m+M) - m\cos^2\theta}$$

$$= \frac{2gh''}{1 - (m\cos^2\theta)/(m+M)}$$

$$v_o = \left[\frac{2gh''}{1 - (m\cos^2\theta)/(m+M)}\right]^{1/2}$$

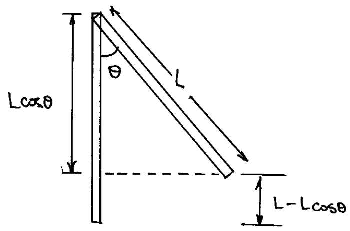
$$= \left[\frac{2g(h - L\sin\theta)}{1 - (m\cos^2\theta)/(m+M)}\right]^{1/2}$$

(b)

$$KE = \frac{1}{2}(m+M)\left(\frac{mv_o\cos\theta}{m+M}\right)^2$$

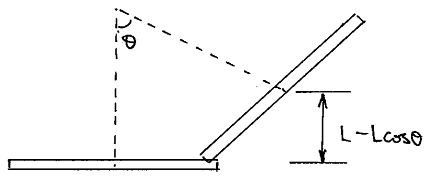
$$= \frac{1}{2}\frac{(mv_o\cos\theta)^2}{m+M}$$

5.



The center of mass is raised by:

$$h_b = \frac{L - L\cos\theta}{2}$$



The center of mass is raised by:

$$h_a = L - L\cos\theta$$

Potential Energy,

$$PE = mg\left[\frac{L - L\cos\theta}{2} + (L - L\cos\theta)\right]$$

$$= \frac{3}{2}mg(L - L\cos\theta)$$

$$= \frac{3}{2}mgL(1 - \cos\theta)$$

$$= \frac{3}{2}mgL\left[1 - \sqrt{1 - \frac{x^2}{L^2}}\right] \approx \frac{3}{2}mgL\left[1 - \left(1 - \frac{x^2}{2L^2}\right)\right] = \frac{3mg}{4L}x^2$$

Kinetic Energy,

$$KE = \frac{17}{24}mv^2$$

PE + KE = Constant

$$\frac{3mg}{4L}x^2 + \frac{17}{24}mv^2 = \text{Constant}$$

(b)

The total energy is equivalent to that of a mass-spring system with an effective mass of $m_{\text{eff}} = \frac{17}{12}m$ and an effective spring constant of $k_{\text{eff}} = \frac{3mg}{4L}$.

Hence

$$\omega = \sqrt{\frac{k_{\text{eff}}}{m_{\text{eff}}}} = \sqrt{\left(\frac{3mg}{2L}\right)\left(\frac{12}{17m}\right)} = \sqrt{\frac{18g}{17L}}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{17L}{18g}}$$

(c)

Assume that the initial velocity is 0. When the initial displacement is x_0, the simple harmonic motion is given by

$$x = x_0 \cos(\omega t)$$

When $x = x_0/2$ for the first time,

$$\frac{x_0}{2} = x_0 \cos(\omega t) \quad \Rightarrow \quad \cos(\omega t) = \frac{1}{2} \quad \Rightarrow \quad \omega t = \frac{\pi}{3} \quad \Rightarrow \quad t = \frac{\pi}{3\omega} = \frac{\pi}{3}\sqrt{\frac{17L}{18g}} \text{ or } t = \frac{T}{6}$$