Hong Kong High School Physics Olympiad 2007
2007 年香港中學物理競賽

Written Examination
筆試

Jointly Organized by

Education and Manpower Bureau
教育統籌局
The Hong Kong Physical Society
香港物理學會
The Hong Kong University of Science and Technology
香港科技大學

共同舉辦

May 27, 2007
2007 年 5 月 27 日

Rules and Regulations 競賽規則

1. All questions are in bilingual versions. You can answer in either Chinese or English.

   所有題目均為中英對照。你可選擇以中文或英文作答。

2. The multiple-choice answer sheet will be collected 1.5 hours after the start of the contest. You can start answering the open-ended questions any time after you have completed the multiple-choice questions

   選擇題的答題紙將於比賽開始後一小時三十分收回。完成選擇題後，你即可開始作答開放題。

3. Your Participant Number is printed on the small green label. Please follow the instructions on the multiple-choice answer sheet, and use a HB pencil to write your 8-dight Participant Number in the space of “I.D. No.”, and fill out the appropriate circles fully. After that, write your English name in the space provided.

   你的參賽號碼印在已派發給你的綠色標簽上。請依照選擇題答題紙的指示，用 HB 鉛筆在選擇題答題紙的 I.D. No.欄上寫上你的 8 位數參賽號碼，**並把印有所填寫的數目字的圓圈完全塗黑**，然後在適當的空格上填上你的英文姓名。

4. On the cover of the answer book, please write your Chinese and English name in the field of “Student Name” and your 8-dight Participant Number in the field of “Student Number”. You can write your answers on both sides of the sheets in the answer book.

   在答題簿封面上，請於 Student Name 欄中填上你的中英文姓名；於 Student Number 填上你的 8 位數參賽號碼。 答題簿可雙面使用。

5. After you have made the choice in answering a multiple choice question, fill the corresponding circle on the multiple-choice answer sheet fully using a HB pencil.
   填寫選擇題答案時，請將選擇題答題紙上相應的圓圈用 HB 鉛筆完全塗黑。

6. The information provided in the text and in the figure of a question should be put to use together.
   解題時要將文字和簡圖提供的條件一起考慮。

7. Some open problems are quite long. Read the entire problem before attempting to solve it. If you cannot solve the whole problem, try to solve some parts of it. You can even use the answers in some unsolved parts as inputs to solve the others parts of a problem.
   開放題較長，最好將整題閱讀完後再著手解題。若某些部分不會做，也可把它們的答案當作已知來做其它部分。

8. The questions with the ‘*’ sign may require information on page-3.
   帶 * 的題可能需要用到第三頁上的資料。

The following symbols and constants are used throughout the examination paper unless otherwise specified:

g – gravitational acceleration on Earth surface, 9.8 (m/s^2)
G – gravitation constant, 6.67×10^{-11} ($N\ m^2/kg^2$)
e – charge of an electron, -1.6×10^{-19} (A s)
ε_0 – electrostatic constant, 8.85×10^{-12} (A s)/(V m)
m_e – electron mass = 9.11×10^{-31} kg
c – speed of light in vacuum, 3.0×10^8 m/s
Radius of Earth = 6378 km
Sun×-Earth distance (= 1 Astronomical Unit (AU)) = 1.5×10^{11} m
Earth-Moon distance = 3.84×10^8 m
Density of water = 1.0×10^3 kg/m^3
Density of iron = 7.7×10^3 kg/m^3
Density of mercury = 13.6×10^3 kg/m^3
Speed of sound in air = 340 m/s

除非特別注明，否則本卷將使用下列符號和常數：

g – 地球表面重力加速度, 9.8 (m/s^2)
G – 萬有引力常數, 6.67×10^{-11} ($N\ m^2/kg^2$)
e – 電子電荷, -1.6×10^{-19} (A s)
ε_0 – 靜電常數, 8.85×10^{-12} (A s)/(V m)
m_e – 電子質量, 9.11×10^{-31} kg
c – 真空光速, 3.0×10^8 m/s
地球半徑 = 6378 km
太陽-地球距離 (= 1 天文單位) = 1.5×10^{11} m
地球-月球距離 = 3.84×10^8 m
水的密度 = 1.0×10^3 kg/m^3
鐵的密度 = 7.7×10^3 kg/m^3
水銀的密度= 13.6×10^3 kg/m^3
空氣中聲速= 340 m/s

The following conditions will be applied unless otherwise specified:

1) All objects are near Earth surface and the gravity is pointing downwards.
2) Neglect air resistance.
3) All speeds are much smaller than the speed of light.

除非特別注明，否則下列條件將適用於本卷所有問題：

1) 所有物體都處於地球表面，重力向下；
2) 忽略空氣阻力；
3) 所有速度均遠小於光速。

Multiple Choice Questions

(2 points each. Select one answer in each question.)

選擇題（每道題 2 分，每道題選擇一個答案）

MC1

It takes 240 s for the escalator to bring a boy, who is standing still, from the bottom to the top. If the boy walks on the moving escalator, it takes 60 s for him to reach the top. If the escalator is not operating, how long does it take for the boy to walk from the bottom to the top?

一運行中的行人電梯從地下到頂樓需時 240 秒（s）。一男孩在運行中的電梯上走，從地下到頂樓需時 60 s。現在電梯停了。問男孩從地下走到頂樓需時多少？

(a) 80 s (b) 140 s (c) 150 s (d) 60 s (e) 120 s

MC2

Ship A moves due north at 40 km/h, while ship B moves due west at 30 km/h. Find the relative speed between the two ships.

船-A 以每小時 40 公里（40 km/h）的速度向北行駛，船-B 以 30 km/h 的速度向西行駛。求兩船之間的相對速度值。

(a) 10 km/h (b) $\frac{30}{\sqrt{2}}$ km/h (c) 50 km/h (d) 40 km/h (e) 60 km/h

MC3

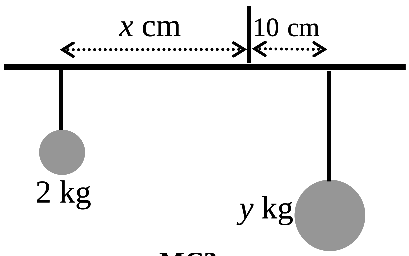
The balance with beam and strings of negligible masses is at equilibrium. Choose the correct values of x and y below.

秤的杆和細繩的質量可忽略。選出下列中正確的 x 和 y 答案。

(a) $x = 1, y = 2$ (b) $x = 10, y = 1$

(c) $x = 20, y = 4$ (d) $x = 40, y = 10$

(e) $x = 2, y = 2$

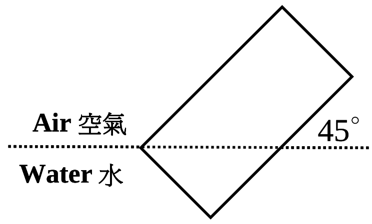


MC3

MC4*

An object of weight W has a uniform rectangular cross-section of $a \times 2a$ and density of 0.25 g/cm^3. Part of it is immersed in water and the rectangle is tilted by 45º, while one of its corners is just at the water surface. Find the torque of the buoyancy force to the center of mass of the object.

一均勻物體重量爲 W，比重爲 0.25 g/cm^3，截面爲 $a \times 2a$ 的長方形，部分浸在水裏，其中一角恰好在水面，長方形與水面成 45 º角。求浮力對物體重心的力矩。



MC4

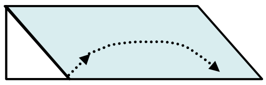
(a) $2\sqrt{2}aW$ (b) $\frac{aW}{\sqrt{2}}$ (c) $\frac{aW}{2}$ (d) $\frac{aW}{2\sqrt{2}}$ (e) aW

MC5*
A dwarf planet, Eris, was discovered recently together with its satellite Dysnomia. The Eris-Dysnomia orbital period is about 14 (Earth) days. Assuming that the mass of Dysnomia is much smaller than Eris, and it performs circular motion with a radius of 33,000 km around Eris, find the mass of Eris.
最近發現的矮行星 Eris，它的衛星 Dysnomia 繞它軌道運動的週期約爲 14 地球日，軌道半徑爲 33,000 km。Dysnomia 的質量比 Eris 小很多。求 Eris 的質量。

(a) 8.5×10^{21} kg (b) 1.45×10^{22} kg (c) 2.45×10^{22} kg (d) 2.85×10^{22} kg

(e) 3.5×10^{21} kg

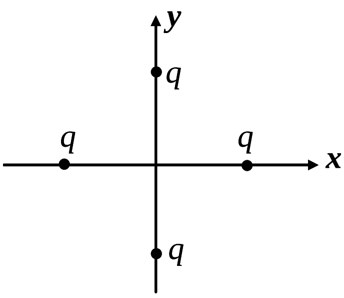
MC6*
A small ball is projected up a smooth inclined plane with an initial speed of 9.8 m/s along the direction at 30° to the bottom edge of the slope. It returns to the edge after 2 s. The ball is in contact with the inclined plane throughout the process. What is the inclination angle of the plane?


MC6

一小球以初速度 9.8 m/s 在一光滑斜面的底邊沿與底邊成 30°的方向貼著斜面射出。小球始終與斜面接觸。2 秒後小球回到底邊。求斜面的傾斜角。

(a) 15° (b) 60° (c) 37° (d) 45° (e) 30°

MC7
Four point charges, each carrying charge q, are at the positions with coordinates of $(a, 0)$, $(-a, 0)$, $(0, a)$, $(0, -a)$, respectively. Find the electric field strength at $(0, a/2)$.
四個電荷爲 q 的點電荷，分別放在坐標爲 $(a, 0)$, $(-a, 0)$, $(0, a)$, $(0, -a)$ 的位置。求 $(0, a/2)$ 處的電場值。

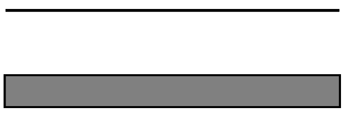


MC7

(a) $\frac{4}{4\pi\varepsilon_0}\frac{q}{a^2}$ (b) $\frac{1}{4\pi\varepsilon_0}\frac{q}{a^2}$ (c) $\frac{3.84}{4\pi\varepsilon_0}\frac{q}{a^2}$

(d) $\frac{1.84}{4\pi\varepsilon_0}\frac{q}{a^2}$ (e) $\frac{2.84}{4\pi\varepsilon_0}\frac{q}{a^2}$

MC8
A parallel-plate capacitor consists of two conductor plates of area A and separated by a distance d. A dielectric slab with dielectric constant ε, thickness $d/4$ and area A is inserted between the plates. What is the capacitance of the capacitor?
一面積爲 A 的平行板電容器，兩板間距離爲 d，板之間的空間有一厚爲 $d/4$、介電常數爲 ε、面積同樣爲 A 的的電介質平板。求電容器的電容。


MC8

(a) $\frac{\varepsilon_0 A}{d}\left(\frac{4\varepsilon}{1+3\varepsilon}\right)$ (b) $\frac{\varepsilon_0 A}{d}\left(\frac{4\varepsilon}{1+\varepsilon}\right)$ (c) $\frac{\varepsilon_0 A}{d}\left(\frac{2\varepsilon}{1+\varepsilon}\right)$

(d) $\frac{\varepsilon_0\varepsilon A}{4d}$ (e) $\frac{\varepsilon_0\varepsilon A}{d}$

MC9*

An electron is in uniform circular motion in a uniform magnetic field perpendicular to the circular orbit. If the period of the circular motion is 1.0×10^{-6} s, what is the magnitude of the magnetic field?

一電子在均勻磁場內作勻速圓周運動，週期爲 1.0×10^{-6} 秒。磁場與圓平面垂直。求磁場大小。

(a) 3.0×10^{-4} T (b) 8.6×10^{-4} T (c) 1.6×10^{-5} T
(d) 2.6×10^{-5} T (e) 3.6×10^{-5} T

MC10*

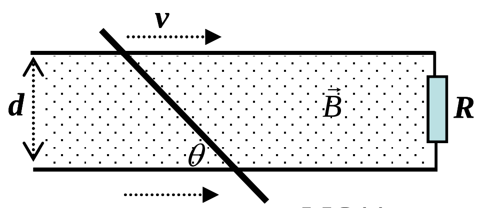
At a certain moment, the passengers on an airplane flying at a height of 8 km see sunrise. How long would it take for the people on the ground directly below the plane to see sunrise?

一架飛機在 8 千米上空飛行。機上乘客剛好見到日出。問飛機垂直下方的人們要過多久才能見到日出？（min. = 分钟）

(a) 5.8 min. (b) 7.3 min. (c) 9.1 min. (d) 11.5 min. (e) 13.2 min.

MC11

A conducting rod with resistance r per unit length is moving inside a vertical magnetic field $\vec{B}$ at speed v on two horizontal parallel ideal conductor rails. The ends of the rails are connected to a resistor R. The separation between the rails is d. The rod maintains a tilted angle θ to the rails. Find the external force required to keep the rod moving.



MC11

兩根平行的理想導電金屬導軌水平放在方向向上的均勻磁場 $\vec{B}$ 中，導軌間距爲 d。導軌端點連著電阻爲 R 的電阻器。一每單位長度電阻爲 r 的導體棒在導軌上以速度 v 滑動，導體棒與導軌保持角度 θ。求保持導體棒運動的外力大小。

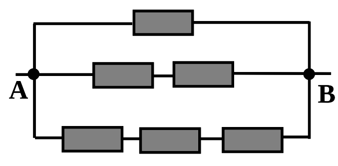
(a) $F = \dfrac{B^2d^2v}{(R+dr)}$ (b) $F = \dfrac{B^2d^2v}{(R+dr/\sin\theta)}$ (c) $F = \dfrac{B^2d^2v/\sin^2\theta}{(R+dr/\sin\theta)}$

(d) $F = \dfrac{B^2d^2v/\cos^2\theta}{(R+dr/\cos\theta)}$ (e) $F = \dfrac{B^2d^2v}{(R+dr/\cos\theta)}$

MC12

The resistance of each resistor is 1Ω. Find the equivalent resistance between points A and B.

每個電阻爲 1Ω。求 A、B 間的等效電阻。



MC12

(a) 6/11Ω (b) 11/6Ω (c) 2/3Ω
(d) 3/4Ω (e) 2Ω

MC13*

A rocket is launched vertically upward from ground and moves at a constant acceleration of 19.6 m/s^2. By accident, the engine is suddenly shut off 10 seconds after launch. To escape, the astronauts must eject at least 3 seconds before the rocket hits the ground. Neglect air resistance. How long will the astronauts have before ejection?

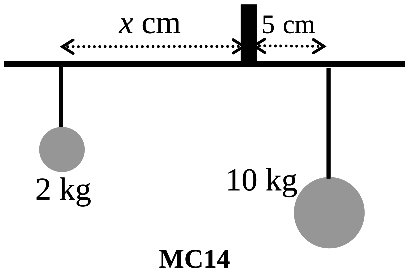
一火箭由地面以 19.6 m/s^2 的加速度垂直發射。由於故障，火箭在 10 秒鐘後突然熄滅。若宇航員必須在火箭撞地前至少 3 秒鐘彈射跳傘，問宇航員還有多少時間準備跳傘？

(a) 45.5 s (b) 34.5 s (c) 6.5 s (d) 41.5 s (e) 13.5 s

MC14

An object of about 10 kg is measured by a not-so-ideal balance. The pivot is provided by a 4.0 mm wide ribbon, so the uncertainty of the position of the pivot is 4 mm. All distances on the scale are measured from the midpoint of the ribbon. Estimate the percentage error if the weight of the object is measured using this balance.

一個不太理想的秤被用來量度一個重量約 10 kg 的物體。秤的支點為一寬 4.0 mm 的布帶，因此支點的位置有 4.0 mm 的不確定性。秤杆上各點的位置均以布帶的中點為參照點。估計所量物體重量的百分誤差。



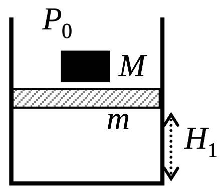
MC14

(a) 0 ~ 1.0 % (b) 1.6 % ~ 2.0 % (c) 8 % ~ 10 % (d) 20 % ~ 30 %
(e) 50 % ~ 60 %

MC15

A piston chamber of cross section area A is filled with ideal gas. At equilibrium a sealed piston of mass m is at the height H_1 from the bottom of the cylinder. The friction force between the chamber wall and the piston can be ignored. The atmosphere pressure is P_0. Now a weight of mass M is added onto the piston. Find the height of the piston. The temperature remains unchanged.

一橫截面積為 A 的圓筒，筒內有一可上下無摩擦滑動且不漏氣的質量為 m 的活塞，活塞下方為理想氣體。平衡時活塞位於離圓筒底 H_1 的位置。已知溫度不變，大氣壓強為 P_0。現在活塞上加一質量為 M 的重物，求氣缸的位置。



MC15

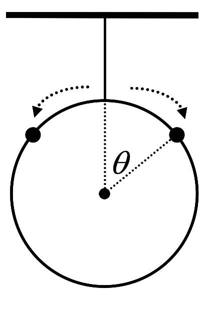
(a) $\dfrac{P_0A+mg}{(M+m)g+P_0A}H_1$ (b) $\dfrac{P_0A}{Mg+P_0A}H_1$ (c) $\dfrac{P_0A-mg}{(M+m)g-P_0A}H_1$

(d) $\dfrac{P_0A+mg}{(M-m)g+P_0A}H_1$ (e) H_1

MC16

A smooth circular track of mass M is vertically hung by a string down the ceiling. Two small rings, each of mass m, are initially at rest at the top of the track. They then slide down simultaneously along the track in opposite directions. Find the position of the rings when the tension in the string is zero.

一質量為 M 的光滑圓圈用細繩垂直掛在天花板上。兩個質量為 m 的小圓環從圈頂由靜止開始同時向兩邊下滑。求當細繩張力為零時小圓環的位置。



MC16

(a) $\theta=\cos^{-1}\left[\dfrac{5}{3}\left(1+\sqrt{m-\dfrac{3M}{2m}}\right)\right]$

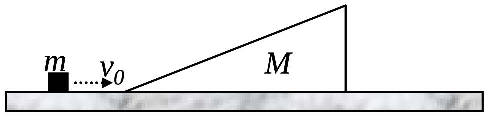
(b) $\theta=\cos^{-1}\left[\dfrac{5}{3}\left(1+\sqrt{1-\dfrac{3m}{2M}}\right)\right]$ (c) $\theta=\cos^{-1}\left[\dfrac{1}{3}\left(1+\sqrt{1-\dfrac{3M}{2m}}\right)\right]$

(d) $\theta=\cos^{-1}\left[\dfrac{1}{3}\left(1+\sqrt{1-\dfrac{5M}{3m}}\right)\right]$ (e) $\theta=\sin^{-1}\left[\dfrac{1}{3}\left(1+\sqrt{1-\dfrac{5m}{3M}}\right)\right]$

MC17

A small block of mass m is moving on a smooth horizontal table surface at initial speed v_0. It then moves smoothly onto a sloped big block of mass M. The big block can also move freely on the table surface. After the small block reaches the height h on the slope, it slides down. Find the height h.

一質量爲 m 的小物塊以初速度 v_0 在光滑水平桌面上滑行。之後它平滑地滑上一質量爲 M，有平滑斜面的大物塊。大物塊可在桌面上自由滑行。到達斜面最高點後小物塊滑下大物塊。求最高點的高度 h。



MC17 & 18

(a) $h=\dfrac{v_0^2}{2g}$, (b) $h=\dfrac{1}{g}\dfrac{Mv_0^2}{m+M}$, (c) $h=\dfrac{1}{2g}\dfrac{mv_0^2}{m+M}$,

(d) $h=\dfrac{1}{2g}\dfrac{Mv_0^2}{m+M}$, (e) $h=\dfrac{v_0^2}{g}$

MC18

Following MC17, find the speed v of the small block after it leaves the slope.

接著 **MC17**，求小物塊離開大物塊後的速率 v。

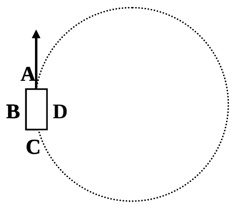
(a) $v=-v_0$ (b) $v=\dfrac{m}{m+M}v_0$ (c) $v=\dfrac{M-m}{m+M}v_0$

(d) $v=\dfrac{m-M}{m+M}v_0$ (e) $v=\dfrac{M}{m+M}v_0$

MC19

A balloon filled with helium gas is tied by a light string to the floor of a car. With all the windows shut, the string remains vertical when the car is moving at a constant velocity. If the car is traveling with constant speed along a circular path, what direction will the string tilt towards?

一充滿氦氣的氣球繫在細繩的上端，細繩的下端固定在密封的車的地板上。當車作勻速運動時，細繩是豎直的。當車作勻速圓周運動時，細繩應向哪方向傾斜？



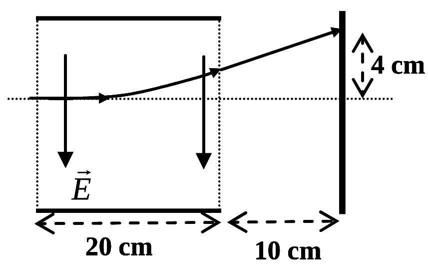
MC19

(a) A (b) B (c) C (d) D (e) Remains vertical 保持豎直

MC20*

An electron is moving at a horizontal velocity of 5×10^6 m/s before it enters a region of uniform downward electric field. It leaves the region after traveling a horizontal distance of 20cm. If the electron is deflected by 4 cm when it reaches a screen 10 cm from the electric field region, find the field magnitude.

一電子以 5×10^6 m/s 的水平初速度進入長爲 20cm 的均勻電場區，區內電場垂直向下。電場區與屏幕之間是一長爲 10 cm 的無場區。若電子的偏離爲 4 cm，求電場強度。



MC20

(a) 142 N/C (b) 102 N/C (c) 213 N/C (d) 42 N/C

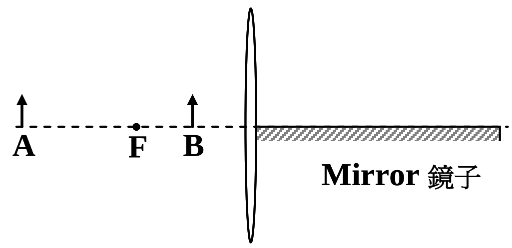
(e) 355 N/C

《**End of MC's** 選擇題完》

Open Problems 開放題
Total 5 problems 共 5 題

Q1 (8 points)

Object-A is at a distance 2*f* from a positive lens of focal length *f*, and Object-B is at *f*/2 from the lens. On the other side of the lens there is a flat mirror laying parallel to and coincide with the optical axis of the lens. Find the positions of the final images and the magnifications of the two objects. Also, determine whether the images are real or virtual.

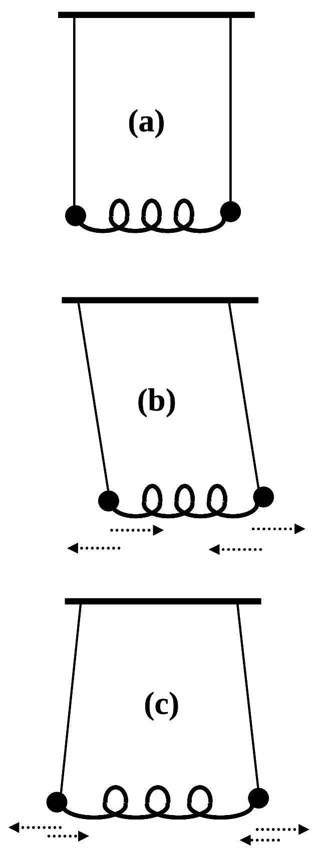


題 1（**8** 分）

一焦距爲 *f* 的正透鏡左邊有兩個物。物-A 離透鏡的距離爲 2*f*，物-B 離透鏡的距離爲 *f*/2。透鏡右邊有一鏡子，鏡面與透鏡光軸平行並與之重合。求兩個物最後的成像位置和放大率，並確定像的虛實。

Q2 (8 points)

Shown in Figure-a are two identical simple pendulums joint by an ideal spring of force constant *k* and natural length *d*, which is equal to the distance between the fixed points of the two pendulums. At equilibrium the two strings (length *L*) attached to the two weights (both having mass *M*) are straight upright. For such a system even the small amplitude simple harmonic motions are quite complicated, but there are two 'normal mode' motions that are relatively simple. One is shown in Figure-b, in which the two weights move in unison, i. e., in the first quarter of the cycle, both weights move to the right by the same amount, and in the third quarter of the cycle, both move to the left. The spring is neither stretched nor compressed in the entire cycle. In the second mode shown in Figure-c, the two weights are moving in opposite directions while keeping their combined center of mass fixed. In the first quarter of the cycle, both move outwards by the same amount, and in the third quarter of the cycle both move inward by the same amount. Find the periods of the two modes.

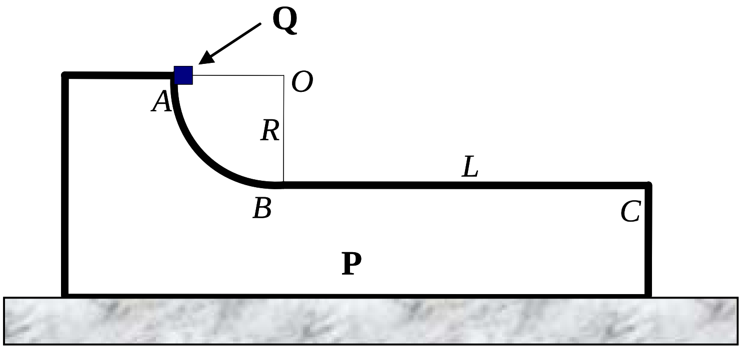


題 2（**8** 分）

如圖-a 所示，兩個相同的單擺，細線長爲 *L*，重物質量爲 *M*，細線上端固定點間的距離爲 *d*。重物間以一理想彈簧相連，彈簧的彈性係數爲 *k*，自然長度與細線上端固定點間的距離相同。平衡時細線與地面垂直。這樣的系統的小幅簡諧振蕩運動也相當複雜，但有兩個相對簡單的 '正則' 振盪模式。模式-1 如圖-b 所示，兩個重物同步運動，在第一個 1/4 週期兩個重物一起向右運動且幅度相同，在第三個 1/4 週期兩個重物一起向左運動且幅度相同。彈簧在整個週期無伸長或壓縮。模式-2 如圖-c 所示，兩個重物運動正好相反，而它們共同的質心保持靜止。在第一個 1/4 週期兩個重物一起向外運動且幅度相同，在第三個 1/4 週期兩個重物一起向裏運動且幅度相同。求這兩個振盪模式的振盪週期。

Q3 (12 points)
On a smooth horizontal ground surface there is a big block P of mass M. Its AB section is 1/4 of a circle of radius R, while its BC section is a horizontal surface of length L. A small cube Q of mass m is released from the top of the arc from rest and slide down. When it reaches point-B at the bottom of the arc its speed relative to P is v. It then continues to move forward and finally stops at point-C.

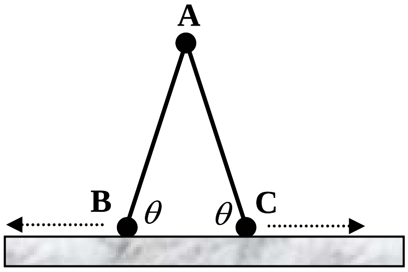
(a) Find the maximum value of v when Q reaches point-B in terms of R, M, m, and g.
(b) Given speed v, find the kinetic friction coefficient between P and Q.
(c) Find the displacement of P relative to the ground when Q reaches point-C.



題 3（**12** 分）
光滑的水平地面上有一質量爲 M 的滑塊 P，其 AB 段爲 l/4 圓弧，半徑爲 R，BC 段是水平面，長爲 L。一質量爲 m 的物塊 Q，從 A 點由靜止釋放，沿 ABC 滑行，到 B 點時 Q 相對與 P 的速度爲 v，最後停在 C 點。

(a) 求物塊 Q 到 B 點時速度 v 的最大可能值。(以 R, M, m, g 表達)
(b) 如果已知速度 v，求物塊 Q 與 BC 面之間的滑動摩擦係數。
(c) 求物塊 Q 到 C 點時滑塊 P 相對與桌面的位移。

Q4 (10 points)
As shown, two weightless and rigid thin rods are connected by a spherical joint A of mass M. The rods can swing freely around the joint. On the other ends of the rods are two small hard balls (B and C) of masses M and $2M$, respectively. Originally both rods are upright on a smooth table surface with joint A on the top, and B and C are on the surface. After releasing, balls B and C remain on the surface and move sideways, while the rods remain in the plane perpendicular to the table surface. (a) Find the velocity of A right before it hits the table surface. (b) Find the velocity of A when the rods are at angle θ to the table surface after releasing from the upright position. (c) Using the results in (b) to verify your answer in (a).

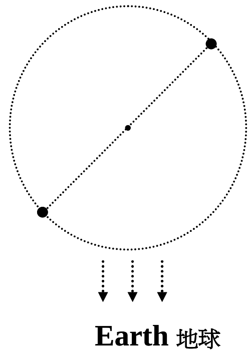


題 4（**10** 分）
兩根長度均爲 L 的剛性輕細杆，一端用質量爲 M 的球型鉸鏈 A 相連，另端分別安裝質量爲 M 和 $2M$ 的小球 B、C。細杆可繞鉸鏈自由轉動。開始時兩杆併攏，鉸鏈 A 在上，豎直放置在光滑桌面上，從靜止釋放，小球 B、C 始終在桌面上向兩邊滑動，兩杆始終在與桌面垂直的平面裏。細杆的質量和各種磨擦均可忽略。(a) 求鉸鏈 A 碰到桌面前瞬間的速度。(b) 求從靜止釋放後，當杆與桌面成角度 θ 時鉸鏈 A 的速度。(c) 用(b)的結果驗證(a)的答案。

Q5* (22 points) 題 5*（22 分）

About half of the stars in the sky are actually binaries, i. e., two stars revolving around a fixed point bound by their mutual gravity. Because the two stars in a binary system are quite close to each other and they are at far distance from us, they look like a single star in the sky. Consider a binary system consisting of two stars of the same mass as our Sun, and the distance between them is 1 Astronomical Unit. Both revolve around a fixed center in a circular orbit, as shown.

星空中大約有一半的星其實是雙星，也就是兩個恒星在相互重力作用下繞一固定點轉動。由於雙星間的距離不大，但離我們很遠，所以看起來像是一顆星。現有兩個質量與太陽相同的恒星組成雙星，雙星間的距離爲 1 天文單位，兩恒星均繞一固定點在圓軌道上轉動。



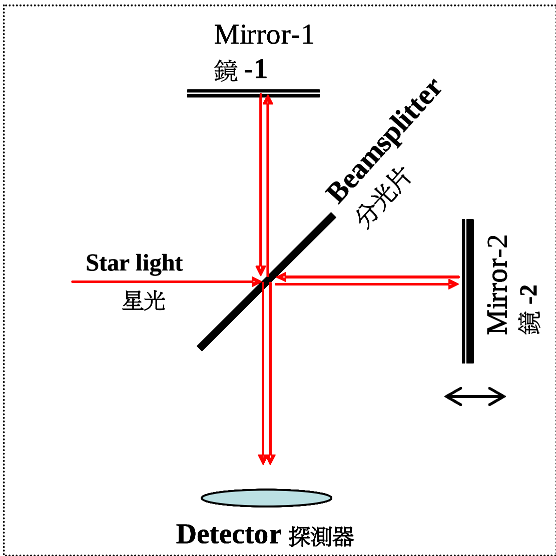
(a) Where is the fixed point? (2 points)
固定點在哪裏？（2 分）

(b) Find the revolving period in the unit of year. (3 points)
求軌道運動的週期（以年爲單位）。（3 分）

(c) Our Earth happens to be in such a position that our line-of-sight is parallel to the orbital plane of the binary. For the light of wavelength λ = 500 nm emitted from the stars, find the maximum Doppler shift $\delta\lambda$, and draw a sketch of the Doppler shift as a function of time. (4 points)

從地球上看，我們的視線與雙星軌道平面恰好平行。雙星發射光的波長爲 λ = 500 nm，求光的最大多普勒頻移，並用簡圖畫出頻移與時間的關係。（4 分）

(d) A Michelson interferometer is employed to detect the Doppler shifts when the separation between the two stars appears largest. The light from the stars entering the instrument is a narrow parallel beam containing two waves of equal intensity I_0 with wavelengths $\lambda - \delta\lambda$ and $\lambda + \delta\lambda$, where $\lambda \gg \delta\lambda$. The beamsplitter reflects half (in intensity) of the incident waves, and let the other half through, without introducing any phase shifts. The optical path distance between Mirror-1 and the beamsplitter is fixed at L while the distance between the beamsplitter and Mirror-2 is $L + x$, where x can be varied. The waves reflected from Mirror-1 and Mirror-2 finally meet at the detector and interfere. The total light intensity received by the detector can be expressed as $I(x) \cong I_0(1 + f(x)\cos(4\pi x/\lambda))$. Find the function $f(x)$, and the value of x_0 when $f(x_0) = f(0)/2$.
(Hint: The total intensity due to the interference of two waves of equal intensity I_0 and equal wavelength is $I = 2I_0(1+\cos\Delta)$, where Δ is the phase difference between the two waves.) (5 points)



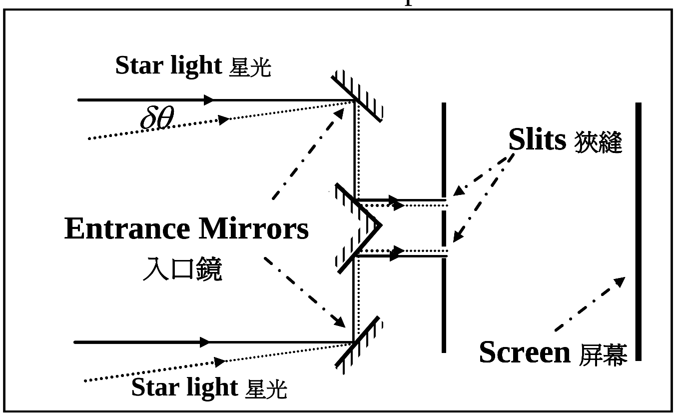
Michelson Interferometer 干涉儀

邁克爾遜干涉儀被用來測量當雙星看上去分開最遠時的多普勒頻移。此時進入干涉儀的星光可當作一束包含兩個強度均爲 I_0，波長分別爲 $\lambda - \delta\lambda$ 和 $\lambda + \delta\lambda$ 的平行光波，其中 $\lambda \gg \delta\lambda$。分光片將一半強度的入射光反射，另一半讓其透射，但不引入位相變化。鏡-1 到分光片的光程爲固定的 L，鏡-2 到分光片的光程爲 $L + x$，其中 x 可調。從兩鏡上反射的光波在探測器相干。探測器測到的總光強可表達成 $I(x) \cong I_0(1 + f(x)\cos(4\pi x/\lambda))$，求涵數 $f(x)$，以及當 $f(x_0) = f(0)/2$ 時 x_0 的值。
（提示：兩個波長相同、強度均爲 I_0 的波干涉所產生的總光強爲 $I = 2I_0(1 + \cos\Delta)$，其中 Δ 爲兩波之間的位相差。）（5 分）

The Doppler Effect can be ignored for the rest of the problem.
題目餘下的部分可忽略多普勒效應。

(e) The binary is at 1000 light-years distance from Earth. Find the maximum angular separation $\delta\theta$ between the two stars in the unit of arc degree. (1 point)
雙星離地球 1000 光年。求雙星最大角距離 $\delta\theta$（用弧度爲單位）。（1 分）

(f) A Stella Interferometer is used to accurately measure the angular separation between the two stars. As shown, the light from the stars can be approximately treated as two broad parallel light waves of 500 nm in wavelength; one (wave-1) is at normal incidence and the other (wave-2) is off by a small angle $\delta\theta$. Each wave is then split into two by the two entrance mirrors. The distance between the mirrors is D. There is no phase difference between the two waves split from wave-1 at the entrances. Find the phase difference between the two waves split from wave-2 at the entrance mirrors. (3 points)



Stella Interferometer 星光干涉儀

星光干涉儀可用來精確測量雙星的角距離。從兩恒星來的波長爲 500 nm 的寬大的平行光波，一束（波-1）正入射，另一束（波-2）與正入射方向成一小偏角 $\delta\theta$。每束波在兩入口鏡被分成兩束，入口鏡之間離距爲 D。從波-1 分出的兩束波在入口鏡處的位相差爲零。求從波-2 分出的兩束波在入口鏡處的位相差。（3 分）

(g) The waves at the entrances are then brought to the two narrow slits of a Young's experiment, without introducing further path differences between the waves through the upper entrance and the ones through the lower entrance. If on the screen the bright interference fringes of one star exactly overlap the dark fringes of the other, what should be the minimum distance between the two entrance mirrors? (In actual operation, the entrance mirrors are moved slowly until the fringes on the screen disappear.) (4 points)

從入口鏡進入干涉儀的光波被引導到 Young's 干涉實驗的狹縫上，從上入口到上狹縫的光程與從下入口到下狹縫的光程相等。若屏幕上一顆星所形成的干涉亮條紋與另一顆星所形成的干涉暗條紋剛好重疊，求兩入口鏡之間的最小距離。（實際操作時，入口鏡被慢慢拉開，直到屏幕上的干涉條紋剛好消失。）（4 分）

《**END** 完》

Answers for M.C. Questions

1) 80s, a; **2)** 50 km/h, c; **3)** x = 20, y = 4, c; **4)** , $\frac{aW}{2\sqrt{2}}$ d; **5)** 1.45 $\times 10^{22}$ kg, b; **6)** 30°, e;

7) $\frac{2.84}{4\pi\varepsilon_0}\frac{q}{a^2}$, e; **8)** $\frac{\varepsilon_0 A}{d}\left(\frac{4\varepsilon}{1+3\varepsilon}\right)$, a; **9)** 3.6 × 10^{-5} T, e; **10)** 11.5 min., d;

11) $F = \frac{B^2 d^2 v}{(R + dr/\sin\theta)}$, b; **12)** 6/11Ω , a; **13)** 41.5 s, d; **14)** 8 % ~ 10 % , c;

15) $\frac{P_0 A + mg}{(M+m)g + P_0 A}H_1$, a; **16)** $\theta = \cos^{-1}\left[\frac{1}{3}\left(1+\sqrt{1-\frac{3M}{2m}}\right)\right]$, c;

17) $h = \frac{1}{2g}\frac{Mv_0^2}{m+M}$, d; **18)** $v_1 = \frac{m-M}{m+M}v_0$, d; **19)** d; **20)** 142 N/C, a

Details

MC 01.

Escalator velocity $= u$, boy velocity $= v$ and L be the length of path

$t_1 = 240s$ and $t_2 = 60s$

$L = ut_1$ and $L = (u+v)t_2$

=> $L = \left(\frac{L}{t_1} + v\right)t_2$

=> $\frac{L}{v} = \frac{t_1 t_2}{t_1 + t_2} = \frac{(240s)(60s)}{(240s - 60s)} = 80s$

MC02.

Simply velocity addition,

$v_{rel} = \sqrt{v_x^2 + v_y^2} = \sqrt{(30kmh^{-1})^2 + (40kmh^{-1})^2} = 50kmh^{-1}$

MC03.

Consider the balance of moment,

$m_1 L_1 = m_2 L_2$

=> $2x = 10y$

=> $x : y = 5:1$

=> The choice is (c). $x = 20$ and $y = 4$

MC04.

$W =$ buoyancy force

Consider the center of mass and symmetry of immersed part of the object,

$r = \frac{1}{4}\sqrt{a^2 + a^2} = \frac{a}{2\sqrt{2}}$

=> $\Gamma = Wr = \frac{aW}{2\sqrt{2}}$

MC05.

$$\frac{GMm}{R^2} = m\omega^2 R \quad => \quad GM = \omega^2 R^3 \quad => \quad M = \frac{\omega^2 R^3}{G}$$

$$\omega = \frac{2\pi}{14 \times 24 \times 3600s} = 5.194 \times 10^{-6} s^{-1}$$

=> $$M = \frac{\omega^2 R^3}{G} = \frac{\left(5.194 \times 10^{-6} s^{-1}\right)^2 \left(3.3 \times 10^7 m\right)^3}{6.67 \times 10^{-11} Nm^{-2} kg^{-2}} = 1.45 \times 10^{22} kg$$

MC06.

Let v_y be the velocity along the plane in the y-direction,

$$v_y = v \sin\theta \qquad \theta = 30^0$$

$$v = u + at \quad => \quad v\sin\theta = -v\sin\theta + (g\sin\phi)t \qquad \phi \text{ is angle of incline}$$

=> $$\sin\phi = \frac{2v\sin\theta}{gt} = \frac{2(9.8ms^{-1})(\sin 30^0)}{(9.8ms^{-2})(2s)} = 0.5$$

=> $$\phi = 30^0$$

MC07.

From the symmetry of charges, we only have to count y-componets,

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{a^2}\left(2 \times \frac{1}{1^2 + 0.5^2}\frac{1}{\sqrt{5}} + \frac{1}{1.5^2} - \frac{1}{0.5^2}\right)$$

$$\vec{E} = -\frac{1}{4\pi\varepsilon_0}\frac{q}{a^2}(2.84)$$

MC08.

Consider the capacitor as two capacitors with different dielectric and thickness in series.

$$C = \frac{\varepsilon_0 A}{d} \quad \text{(parallel plate capacitor)}$$

=> $$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \quad \text{where} \quad C_1 = \frac{\varepsilon_0 A}{(3/4)d} \quad \text{and} \quad C_1 = \frac{\varepsilon\varepsilon_0 A}{(1/4)d}$$

=> $$C = \frac{\varepsilon_0 A}{d}\left(\frac{4\varepsilon}{1 + 3\varepsilon}\right)$$

MC09.

$$qvB = m\omega^2 r \quad \text{and} \quad v = \omega r$$

=> $$B = \frac{\omega m}{q} \quad \text{and} \quad \omega = \frac{2\pi}{T} = 6.28 \times 10^6 s^{-1}, m_e = 9.11 \times 10^{-31} kg, e = 1.6 \times 10^{-19} C$$

=> $$B = 3.6 \times 10^{-5} T$$

MC10.

Radius of earth $R_e = 6378km$

$$\cos\theta = \frac{R_e}{R_e + h} = \frac{6378km}{6378km + 8km} = 0.9987$$

=> $$\theta = 2.87^0$$

$$T_{delay} = (24 \times 60 \min)\frac{2.87^0}{360} = 11.45 \min$$

MC11.

Resistance of rod $= r\dfrac{d}{\sin\theta}$

$$\xi = \left|\frac{d\phi}{dt}\right| = Bdv$$

$$P = \frac{\xi^2}{R_{total}} = \frac{B^2 d^2 v^2}{r(d/\sin\theta) + R} = Fv$$

=> $F = \dfrac{B^2 d^2 v}{r(d/\sin\theta) + R}$

MC12.

Simply consider the resistances in series and parallel,

$\dfrac{1}{R} = \dfrac{1}{1\Omega} + \dfrac{1}{2\Omega} + \dfrac{1}{3\Omega}$ => $R = \dfrac{6}{11}\Omega$

MC13.

At the point the engine just shutting down,

$$s = \frac{1}{2}at^2 = \frac{1}{2}(19.6ms^{-2})(10s)^2 = 980m$$

$$v = at = (19.6ms^{-2})(10s) = 196ms^{-1}$$

Then, consider the free falling,

$$s = ut + \frac{1}{2}at^2$$

$-980 = (196)t + \dfrac{1}{2}(-9.8)t^2$ => $4.9t^2 - 196t - 980 = 0$

=> $t = 44.5s$ or $-171.5s$ (rejected)

=> The time left is 41.5 s.

MC14.

Find $x = 25cm$

Consider the max uncertainty at extreme case as

$m_{\max} = \dfrac{25.4cm}{4.6cm}(2kg) = 11.04kg$ =>

$$\%error = \frac{11.04 - 10}{11.04} \times 100\% = 9.4\%$$

$m_{\min} = \dfrac{24.6cm}{5.4cm}(2kg) = 9.11kg$ =>

$$\%error = \frac{10 - 9.11}{9.11} \times 100\% = 9.8\%$$

The % error is about 10%.

MC15.

According to Boyle's law,

$P_1V_1 = P_2V_2$ => $P_1H_1 = P_2H_2$

$$\left(P_0 + \frac{mg}{A}\right)H_1 = \left(P_0 + \frac{(m+M)g}{A}\right)H_2$$

=> $$H_2 = \frac{P_0A + mg}{(m+M)g + P_0A}H_1$$

MC16.

Balance of force in vertical direction,

$$Mg + 2mg\cos^2\theta = 2N\cos\theta$$

Normal force component,

$$\frac{mv^2}{R} = N$$

Conservation of energy,

$$mgR(1-\cos\theta) = \frac{1}{2}mv^2$$

=> $$Mg + 2mg\cos^2\theta = 4mg\cos\theta(1-\cos\theta)$$

$$6m\cos^2\theta - 4m\cos\theta + Mg = 0$$

$$\cos\theta = \frac{4m \pm \sqrt{16m^2 - 24mM}}{12m}$$

=> $$\cos\theta = \frac{1}{3} \pm \frac{1}{3}\sqrt{1 - \frac{3}{2}\frac{M}{m}}\ .$$

MC17.

To find h, consider at the highest point, mass m and M moving with the same velocity. Then, by conservation of momentum,

$mv_0 = (m+M)V$ => $V = \dfrac{m}{m+M}v_0$.

By conservation of energy,

$$\frac{1}{2}mv_0^2 = \frac{1}{2}(m+M)V^2 + mgh$$

=> $$\frac{1}{2}mv_0^2 = \frac{1}{2}(m+M)\left(\frac{m}{m+M}v_0\right)^2 + mgh$$

=> $$h = \frac{v_0^2}{2g}\left(\frac{M}{m+M}\right).$$

MC18.

Similarly for finding v,

$mv_0 = mv + MV$ => $V = \dfrac{m}{M}(v_0 - v)$ and $\dfrac{1}{2}mv_0^2 = \dfrac{1}{2}mv^2 + \dfrac{1}{2}MV^2$

=> $$v = v_0\left(\frac{m-M}{M+m}\right).$$

MC19. (D) It is not towards 'B' as would be 'obvious'. The balloon experiences a buoyancy force by the air which is opposite to 'gravity'. Inside the car, the 'gravity' component due to centrifugal force points outwards to 'B'.

MC20.

Consider the cases as inside E-field and outside E-field,

Inside the E-field $t_1 = \frac{d}{v} = \frac{20cm}{5\times10^6 ms^{-1}} = 4\times10^{-8} s$

Outside the E-filed $t_2 = 2\times10^{-8} s$

Given $a = \frac{Eq}{m}$, $y_1 = \frac{1}{2}\frac{Eq}{m}t_1^2$, $y_2 = \left(\frac{Eq}{m}t_1\right)t_2$

=> $y = y_1 + y_2 = \frac{Eq}{m}t_1\left(\frac{t_1}{2} + t_2\right)$

Given $y = 4cm$, $q = 1.6\times10^{-19} C$, $m_e = 9.11\times10^{-31} kg$

=> $E = 142NC^{-1}$

Answers for Open Questions

Q1. (8 points)

For Object A,

$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$ => $\frac{1}{2f}+\frac{1}{v}=\frac{1}{f}$ (1 point)

=> $v=2f$ (1 point)

and $m=\left|\frac{v}{u}\right|=1$ (1 point)

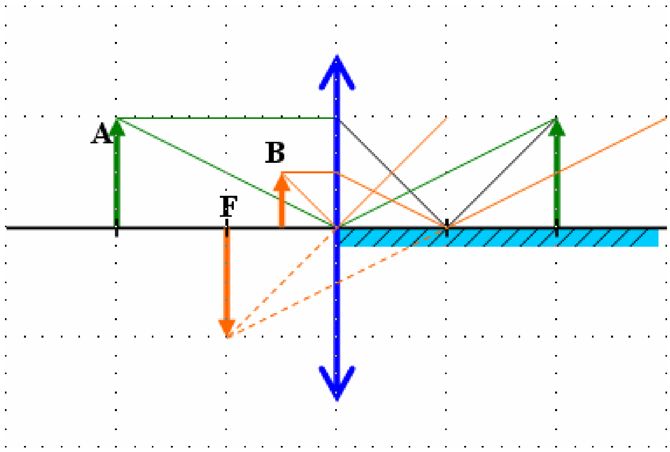
=> real, up (1 point)

For Object B,

$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$ =>

$\frac{1}{f/2}+\frac{1}{v}=\frac{1}{f}$ (1 point)

=> $v=-f$ (1 point)



and

$m=\left|\frac{v}{u}\right|=2$ (1 point)

=> virtual, inverted (1 point)

Q2. (8 points)

For the first mode (Figure b), it is exactly the same as the simple pendulum. (1 point)

$\omega=\sqrt{\frac{g}{L}}$ => $T=2\pi\sqrt{\frac{L}{g}}$ (2 points)

For the second mode (Figure c),

Consider the spring constants contributed from different parts,

From the pendulum,

$k_1=\frac{mg}{L}$ (1 point)

From the spring,

$k_2=2k$ (1 point)

$k_{eff}=k_1+k_2=\frac{mg}{L}+2k=\frac{mg+2kL}{L}$ (1 point)

=> $T=2\pi\sqrt{\frac{m}{k_{eff}}}=2\pi\sqrt{\frac{mL}{mg+2kL}}$ (2 points)

Q3. (12 points)

(a) Consider the arc AB is smooth,

By conservation of momentum,

$mv_1 = MV_1$ => $V_1 = \frac{m}{M}v_1$ (1 point)

By conservation of energy,

$\frac{1}{2}mv_1^2 + \frac{1}{2}MV_1^2 = mgR$ (1 point)

=> $\frac{1}{2}mv_1^2 + \frac{1}{2}M\left(\frac{m}{M}v_1\right)^2 = mgR$

=> $v_1 = \sqrt{2gR}\sqrt{\frac{M}{m+M}}$ and $V_1 = \frac{m}{M}\left(\sqrt{2gR}\sqrt{\frac{M}{m+M}}\right)$

=> $v = v_1 + V_1 = \sqrt{2gR}\sqrt{\frac{M}{m+M}}\left(1+\frac{m}{M}\right) = \sqrt{2gR}\sqrt{1+\frac{m}{M}}$ (2 points)

(b) At the end point, both m and M stopped simultaneously,

By conservation of energy,

$\frac{1}{2}m\left(\frac{M}{m+M}v\right)^2 + \frac{1}{2}M\left(\frac{m}{m+M}v\right)^2 = \mu mgL$ (3 points)

=> $\mu = \left(\frac{M}{m+M}\right)\frac{v^2}{2gL}$ (1 point)

(c) The displacement of P is

$-u^2 = 2aL'$ (1 point) => $-2gR\frac{m}{M}\left(\frac{M}{m+M}\right) = 2\left(-\frac{R}{L}g\right)L'$ (1 point)

(Or because the center of mass of the system remains fixed) (2 points)

$L' = \left(\frac{m}{m+M}\right)L$ (2 points)

Q4. (10 points)

(a) Since the velocity of B and C are zero at the moment when A hits the table, by conservation of energy,

$MgL = \frac{1}{2}Mv^2$ (1 point) => $v^2 = 2gL$ => $v = \sqrt{2gL}$ (1 point)

Method-1 for part-b

According to the condition, B will move to the left at v_B, C will move to the right at v_C, while A will move at $\vec{v}_A = v_{Ax}\vec{x}_0 + v_{Ay}\vec{y}_0$, where x-direction is horizontal and y-direction is vertical.

(1 point)

Energy conservation: $mgL(1-\sin\theta) = \frac{1}{2}m(v_{Ax}^2 + v_{Ay}^2) + \frac{1}{2}mv_B^2 + mv_C^2$ (1) (1 point)

Horizontal momentum conservation: $v_{Ax} + v_B = 2v_C$ (2) (1 point)

The rods are rigid, so the relative velocity between A and B and between A and C along the rod direction must be zero. This leads to two equations below.

$-v_{Ax}\cos\theta + v_{Ay}\sin\theta = v_C\cos\theta$ (3) (1 point)

$v_{Ax}\cos\theta + v_{Ay}\sin\theta = v_B\cos\theta$ (4) (1 point)

(4) – (3) leads to $-2v_{Ax} = v_B - v_C$ (5).

From Eq. (2) and (5) we get $3v_{Ax} = v_C$, and $5v_{Ax} = v_B$. (1 point)

Put in Eq. (4) we get $4v_{Ax}\cos\theta = v_{Ay}\sin\theta$. Put v_{Ax}, v_B and v_C into Eq. (1), we get

$v_{Ax}^2 = \frac{(1-\sin\theta)\sin^2\theta}{44\sin^2\theta + 16\cos^2\theta}2gL$, and $v_{Ay}^2 = \frac{(1-\sin\theta)\cos^2\theta}{11\sin^2\theta + 4\cos^2\theta}2gL$.

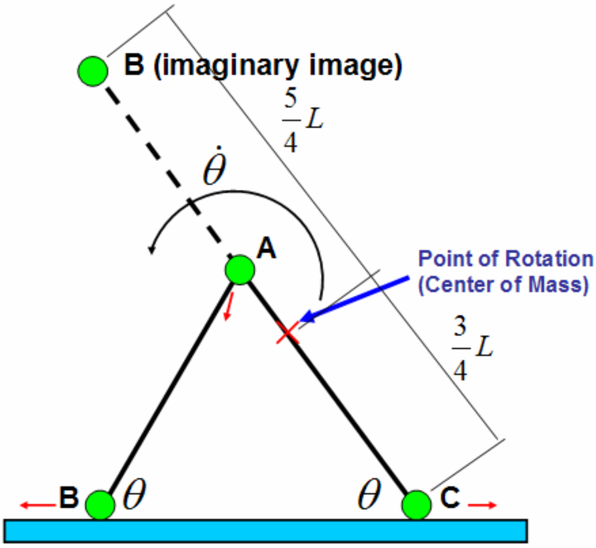
Finally, $v_A^2 = \frac{(1-\sin\theta)(\sin^2\theta + 16\cos^2\theta)}{44\sin^2\theta + 16\cos^2\theta}2gL$ (1 point)

Method-2 for part-b

(b) Find the lateral shifting of the position of A first relative to the center of mass which remains fixed in the horizontal direction,

$$M_A(x-L) + M_B(x-2L) + M_C(x) = 0$$

=> $M(x-L) + M(x-2L) + 2M(x) = 0$ => $x = \frac{3}{4}L$



To find the x-component of velocity of each ball, consider the rotation axis at Center of Mass, while the y-component of velocity of ball A requires to consider the rotation axis at C. Then, we can express the velocity of each mass in term of angular velocity,

$$v_{Ax} = \frac{1}{4}L\dot{\theta}\sin\theta, \qquad v_{Ay} = L\dot{\theta}\cos\theta,\ v_B = \frac{5}{4}L\dot{\theta}\sin\theta,\ v_C = \frac{3}{4}L\dot{\theta}\sin\theta$$

By conservation of energy,

$$mgL(1-\sin\theta) = \frac{1}{2}M_A v_A^2 + \frac{1}{2}M_B v_B^2 + \frac{1}{2}M_C v_C^2$$

=> $$mgL(1-\sin\theta) = \frac{1}{2}m\left(v_{Ax}^2 + v_{Ay}^2\right) + \frac{1}{2}mv_B^2 + \frac{1}{2}(2m)v_C^2$$

=> $$2gL(1-\sin\theta) = L^2\dot{\theta}^2\left[\frac{1}{16}\sin^2\theta + \cos^2\theta + \frac{25}{16}\sin^2\theta + \frac{18}{16}\sin^2\theta\right]$$

$$= L^2\dot{\theta}^2\left[\frac{11}{4}\sin^2\theta + \cos^2\theta\right] = L^2\dot{\theta}^2\left[\frac{7}{4}\sin^2\theta + 1\right]$$

=> $$\dot{\theta}^2 = \frac{8g}{L}\left(\frac{1-\sin\theta}{4\cos^2\theta + 11\sin^2\theta}\right)$$

=> $$v_A^2 = v_{Ax}^2 + v_{Ay}^2 = L^2\dot{\theta}^2\left[\frac{\sin^2\theta}{16} + \cos^2\theta\right] = \frac{gL}{2}(1-\sin\theta)\left(\frac{\sin^2\theta + 16\cos^2\theta}{11\sin^2\theta + 4\cos^2\theta}\right)$$

=> $$v_A = \sqrt{\frac{gL}{2}(1-\sin\theta)\left(\frac{\sin^2\theta + 16\cos^2\theta}{11\sin^2\theta + 4\cos^2\theta}\right)}$$

(c) Put $\theta = 0^0$ =>` $v_A = \sqrt{2gL}$ (1 point)

Q5. (22 points)

(a) Center of mass

(b) The force on one of the star is $\frac{GM_{Sun}^2}{R^2}$, where R is 1 AU. The acceleration is $\omega^2 R/2$

Therefore $M_{sun}\omega^2 \frac{R}{2} = \frac{GM_{Sun}^2}{R^2}$

=> $\omega = \sqrt{\frac{2GM_{Sun}}{R^3}}$ => $\omega = \sqrt{2}\omega_0$, where ω_0 is the orbital frequency of Earth around Sun, and $T_0 \equiv \frac{2\pi}{\omega_0} = 1$ year.

=> $T = 0.71$ years

(c) $v = \left(7.5\times10^{10}\,m\right)\left(2.8\times10^{-7}\,s\right) = 2.1\times10^{4}\,ms^{-1}$

$$\delta\lambda = \lambda\left(\frac{v}{c}\right) = \left(500nm\right)\left(\frac{2.1\times10^{4}\,ms^{-1}}{3\times10^{8}\,ms^{-1}}\right) = 0.035nm$$

The graph of Doppler shifts versus time is sinusoidal.

(d)
$$I(x) = \frac{I_0}{2}\left(1+\cos\left(\frac{4\pi x}{\lambda-\delta\lambda}\right)\right) + \frac{I_0}{2}\left(1+\cos\left(\frac{4\pi x}{\lambda+\delta\lambda}\right)\right)$$

$$= I_0 + \frac{1}{2}I_0\left[\cos\left(\frac{4\pi x}{\lambda-\delta\lambda}\right) + \cos\left(\frac{4\pi x}{\lambda+\delta\lambda}\right)\right]$$

$$= I_0 + I_0\cos\left[2\pi x\left(\frac{1}{\lambda-\delta\lambda}+\frac{1}{\lambda+\delta\lambda}\right)\right]\cos\left[2\pi x\left(\frac{1}{\lambda-\delta\lambda}-\frac{1}{\lambda+\delta\lambda}\right)\right]$$

$$= I_0\left[1+\cos\left[2\pi x\left(\frac{2\lambda}{\lambda^2-\delta\lambda^2}\right)\right]\cos\left[2\pi x\left(\frac{2\delta\lambda}{\lambda^2-\delta\lambda^2}\right)\right]\right]$$

$$\approx I_0\left[1+\cos\left(\frac{4\pi x}{\lambda}\right)\cos\left(\frac{4\pi x}{\lambda}\frac{\delta\lambda}{\lambda}\right)\right]$$

=> $f(x) = \cos\left(\frac{4\pi x}{\lambda}\frac{\delta\lambda}{\lambda}\right)$ $\quad$ $f(0) = 1$

=> $f(x_0) = \frac{f(0)}{2}$ => $x_0 = \cos^{-1}\left(\frac{1}{2}\right)\left(\frac{\lambda^2}{4\pi\delta\lambda}\right) = \frac{1}{12}\frac{\lambda^2}{\delta\lambda} = 0.59$ mm

(e) 1 light year $= 365\times24\times3600s\times\left(3\times10^{8}\,ms^{-1}\right) = 9.46\times10^{15}\,m$

$$\theta = \frac{1.5\times10^{11}m}{1000\times9.46\times10^{15}m} = 1.6\times10^{-8}rad$$

(f) $\Delta x = D\sin\delta\theta$ => $\Delta\theta = \frac{2\pi D\sin\delta\theta}{\lambda} \approx \frac{2\pi D\delta\theta}{\lambda}$

(g) $\Delta\theta = \frac{2\pi D\delta\theta}{\lambda} = \pi$ => $D = \frac{\lambda}{2}\frac{1}{\delta\theta} = 1.56m$