Hong Kong High School Physics Olympiad 2006
2006 年香港中學物理競賽
Written Examination
筆試

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The Hong Kong University of Science and Technology
香港科技大學

共同舉辦

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The following symbols and constants will be used throughout the examination paper unless otherwise specified:

g – gravitational acceleration on Earth surface, 9.8 (m/s^2)
G – gravitation constant, 6.67 x 10^{-11} (N m^2/kg^2)
e – charge of an electron, –1.6 x 10^{-19} (A s)
ε_0 – electrostatic constant, 8.85 x 10^{-12} (A s)/(V m)
m_e – electron mass = 9.11 x 10^{-31} kg
c – speed of light in vacuum, 3.0 x 10^8 m/s
Radius of Earth = 6378 km
Sun-Earth distance = 1.5 x 10^{11} m
Earth-Moon distance = 3.84 x 10^8 m
Density of water = 1.0 x 10^3 kg/m^3
Density of iron = 7.7 x 10^3 kg/m^3
Density of mercury = 13.6 x 10^3 kg/m^3
Speed of sound in air = 340 m/s

除非特別注明，否則本卷將使用下列符號和常數：

g – 地球表面重力加速度, 9.8 (m/s^2)
G – 重力常數, 6.67 x 10^{-11} (N m^2/kg^2)
e – 電子電荷, –1.6 x 10^{-19} (A s)
ε_0 – 靜電常數, 8.85 x 10^{-12} (A s)/(V m)
m_e – 電子質量, 9.11 x 10^{-31} kg
c – 真空光速, 3.0 x 10^8 m/s
地球半徑 = 6378 km
太陽-地球距離 = 1.5 x 10^{11} m
地球-月球距離 = 3.84 x 10^8 m
水的密度 = 1.0 x 10^3 kg/m^3
鐵的密度 = 7.7 x 10^3 kg/m^3
水銀的密度= 13.6 x 10^3 kg/m^3
空氣中聲速= 340 m/s

The following conditions will be applied unless otherwise specified:

1) All objects are near Earth surface and the gravity is pointing downwards.
2) Neglect air resistance.
3) All speeds are much smaller than the speed of light.

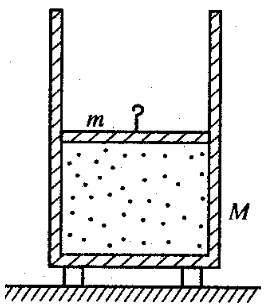
除非特別注明，否則下列條件將適用於本卷所有問題：

1) 所有物體都處於地球表面，重力向下；
2) 忽略空氣阻力；
3) 所有速度均遠小於光速。

Multiple Choice Questions (2 points each. Select one answer in each question.)
選擇題（每道題 2 分，每道題選擇一個答案）

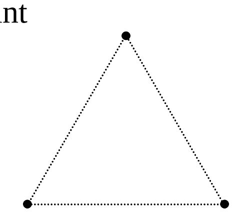
MC1
Given that the moon revolves around Earth at a period of about 27.3 days, use the parameters given in page-2 to find the mass of Earth.
利用**頁 2** 提供的資料，以及月亮大約每 27.3 天繞地球轉一次，估算地球的質量。
(a) 2.3×10^{24} kg (b) 4.6×10^{24} kg (c) 6.0×10^{24} kg (d) 7.8×10^{24} kg
(e) 9.8×10^{24} kg

MC2
As shown, a piston chamber of cross section area A is filled with ideal gas. A sealed piston of mass m is right at the middle height of the cylinder at equilibrium. The friction force between the chamber wall and the piston can be ignored. The mass of the rest of the chamber is M. The atmosphere pressure is P_0. Now slowly pull the piston upwards, find the maximum value of M such that the chamber can be lifted off the ground. The temperature remains unchanged.
如圖所示，在地面上放置一橫截面積爲 A 的圓筒。筒內有一可上下無摩擦滑動且不漏氣的質量爲 m 的活塞，活塞下方爲理想氣體。平衡時活塞正好位於容器內的正中間位置。已知溫度不變，大氣壓強爲 P_0。現在用力非常緩慢地上提活塞，最後要能將氣缸提離地面，求氣缸的質量 M 的最大值。



(a) $M = \dfrac{P_0 A - 2mg}{2g}$ (b) $M = \dfrac{P_0 A - mg}{2g}$ (c) $M = \dfrac{P_0 A - mg}{g}$
(d) $M = \dfrac{P_0 A}{2g}$ (e) $M = \dfrac{P_0 A}{g}$

MC3
Find the electrostatic energy of an equilateral triangle of side a with a point charge q at each vertex, excluding the self-energy of the point charges.
邊長爲 a 的等邊三角形的每個角上有一點電荷 q。求系統的靜電能量（不包括點電荷本身的能量）。



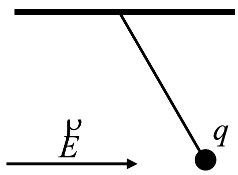
(a) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{q^2}{a}$ (b) $\dfrac{3}{4\pi\varepsilon_0}\dfrac{q^2}{a}$ (c) $\dfrac{3}{2\pi\varepsilon_0}\dfrac{q^2}{a}$
(d) $\dfrac{1}{8\pi\varepsilon_0}\dfrac{q^2}{a}$ (e) $-\dfrac{1}{4\pi\varepsilon_0}\dfrac{q^2}{a}$

MC4
Two cars A and B are moving towards each other along the same line at 2/3 of the sound speed. Car-A sends out continuous sound waves at frequency f. The frequency of sound heard by the driver of car-B is _______.
兩車 A 和 B 各以 2/3 聲速的速度沿直線迎面駛近。A 車發出的聲波的頻率爲 f。B 車聽到的聲波的頻率爲_______。
(a) $5f$ (b) $9f$ (c) $f/5$ (d) $f/9$ (e) f

MC5

As shown, the tiny ball at the end of the thread of length 100 cm has a mass of 0.6 g and carries charge 5.88 x 10^{-6} C. It is in a horizontal electric field of intensity 1000 N/C (Newton per Coulomb). Find the vibration frequency of the ball near its equilibrium position.

如圖，一質量爲 0.6 g 的小球，帶電 5.88 x 10^{-6} C，繫在長爲 100 cm 的細繩一端，在強度爲 1000 N/C 的水平電場裏。求小球在平衡點附近的振動頻率。



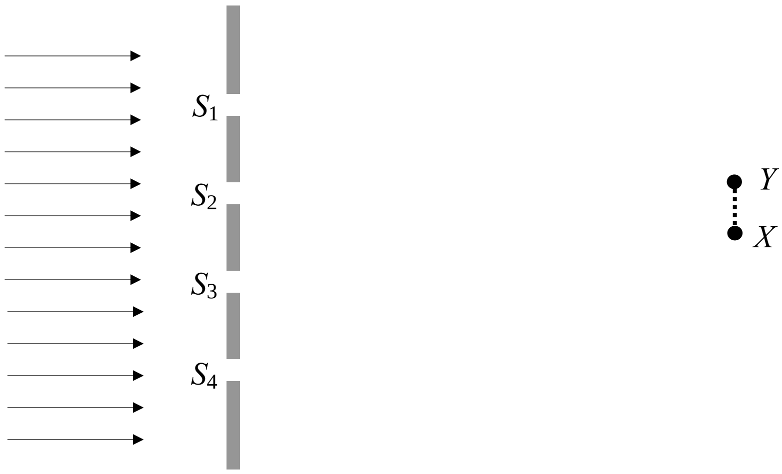
(a) 0.70 Hz (b) 0.59 Hz (c) 0.50 Hz (d) 3.2 Hz
(e) 6.4 Hz

MC6

As shown below, a plane microwave is normally incident on four evenly spaced identical narrow slits S_1, S_2, S_3 and S_4. XY is a line parallel to the plane of the four slits and far away from the slits. Point-X is at equal distance to slits S_2 and S_3. When only slits S_2 and S_3 are open, the wave intensity at point-X reaches the maximum value of A, and that at point-Y is zero. When all four slits are open, the intensity at point-X is _______.

如下圖，一平面微波正入射在四個等距窄縫 S_1、S_2、S_3、S_4 上。XY 連線離窄縫很遠，與窄縫平面平行。點 X 離 S_2、S_3 距離相等。當只有 S_2、S_3 開著時 X 點的波強度達到最大值 A，Y 點的波強度爲零。當四個窄縫都開時，點 X 處的波強度爲 _______ 。

(a) 0 (b) $2A$ (c) $4A$ (d) $8A$ (e) $16A$



MC7

Same condition as MC6. When all four slits are open, the intensity at point-Y is _______.

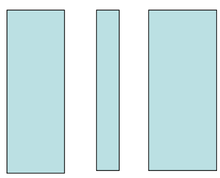
同上題。當四個窄縫都開時，點 Y 處的波強度爲 _______ 。

(a) 0 (b) $2A$ (c) $4A$ (d) $8A$ (e) $16A$

MC8

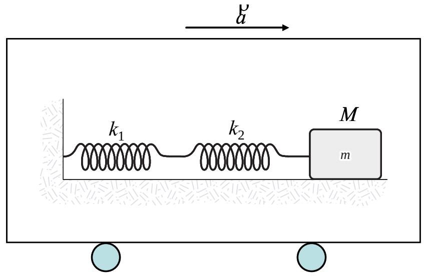
As shown, three large conductor plates of area A are placed parallel to one another at equal distance d. Find the capacitance between the left and the right plates.

如圖，三個大導電平板，面積爲 A，以等間距 d 排列。求左右板之間的電容。



(a) $\frac{\varepsilon_0 A}{d}$ (b) $\frac{\varepsilon_0 A}{2d}$ (c) $\frac{2\varepsilon_0 A}{d}$ (d) $\frac{\varepsilon_0 A}{4d}$ (e) $\frac{4\varepsilon_0 A}{d}$

MC9



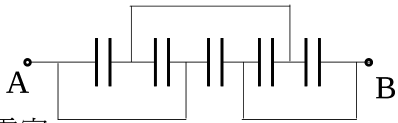
As shown, inside a cart that is accelerating horizontally at acceleration $\vec{a}$ there is a block of mass M connected to two light springs of force constants k_1 and k_2. The block can move without friction horizontally. Find the vibration frequency of the block.

如圖，在一以加速度$\vec{a}$沿水平加速的車廂裏有一個質量爲 M 的物塊。物塊繫在兩根力常數分別爲 k_1、k_2 的輕彈簧上，可在水平面上無磨擦滑動。求物塊的振動頻率。

(a) $\frac{1}{2\pi}\sqrt{\frac{k_1+k_2}{M}}$　　(b) $\frac{1}{2\pi}\sqrt{\frac{k_1+k_2}{M}+a}$　　(c) $\frac{1}{2\pi}\sqrt{\frac{k_1+k_2}{M}-a}$

(d) $\frac{1}{2\pi}\sqrt{\frac{k_1k_2}{(k_1+k_2)M}}$　　(e) $\frac{1}{2\pi}\sqrt{\frac{k_1k_2}{(k_1+k_2)M}+a}$

MC10

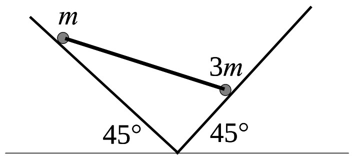


Five identical 1 μF capacitors are connected as shown, find the capacitance between point-A and point-B.

如圖。五個 1 μF 的電容器以圖示方式連接。求點 A 和 B 間的電容。

(a) 1 μF　(b) 2 μF　(c) 3 μF　(d) 4 μF　(e) 5 μF

MC11

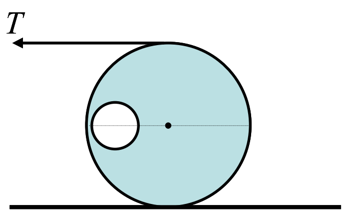


Two small balls of mass m and $3m$, respectively, are connected by a thin and rigid bar with negligible mass, and are free to slide on the 45° inclines, as shown. Find the angle of the bar to the horizontal plane in equilibrium. The angle being negative means that the heavy ball is above the light ball.

兩小球，質量分別爲 m 和 $3m$，由一輕細硬杆連接，可在兩相交的 45°平滑斜面上自由滑行。求當平衡時細杆與水平面的夾角。重球在上夾角爲負。

(a) 46.6°　(b) 26.6°　(c) 45°　(d) – 26.6°　(e) – 46.6°

MC12



A uniform cylinder of radius a originally had a weight of 80 N. After an off-axis cylinder hole at $2a/3$ was drilled through it as shown, it weighs 65 N. The axes of the two cylinders are parallel and their centers are at the same height. A force T is applied to the top of the cylinder horizontally. The value of the force should be _____ in order to keep the cylinder is at rest.

一均勻質量圓柱原重量爲 80 N，半徑爲 a。現在離軸心 $2a/3$ 處打一圓洞，洞軸與大圓柱軸心平行，並在同一高度。圓柱此時重量爲 65 N。在柱頂施一水平方向的力 T。如要柱體平衡，則力的大小爲____。

(a) 1 N　(b) 3 N　(c) 5 N　(d) 8 N　(e) 10 N

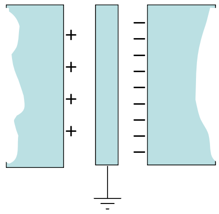
MC13

The electric field is $\vec{E} = (5\vec{x}_0 - 3\vec{y}_0)$ V/m. The potential at the point (0, 5 m, 5 m) is ___ if the potential at coordinate origin is taken as zero.

一電場爲 $\vec{E} = (5\vec{x}_0 - 3\vec{y}_0)$ V/m。如座標原點電勢爲零，則在 (0, 5 m, 5 m) 處的電勢爲 ____。

(a) – 25 V (b) – 15 V (c) 0 (d) 15 V (e) 25 V

MC14



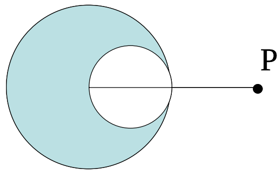
As shown, an infinitely large surface on the left carries fixed surface charge density σ. The one on the right carries -2σ. A conductor slab is inserted between the two and its potential is fixed at zero. In equilibrium, the surface charge densities on the left and the right surfaces of the middle conductor are__________.

如圖，左邊的無窮大平面的固定表面電荷密度爲σ，右邊的無窮大平面的固定表面電荷密度爲 -2σ。現在兩面之間插入一導體板，其電勢保持爲零。平衡時導體板左、右表面的電荷密度分別爲_______。

(a) σ and -σ (b) -2σ and 2σ (c) -2σ and σ

(d) σ and -2σ (e) -σ and 2σ

MC15



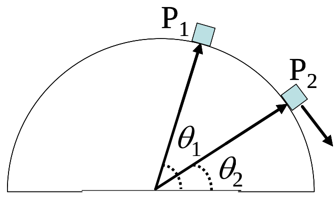
A spherical cave of radius $R/2$ was carved out from a uniform sphere of radius R and original mass M. The center of the cave is at $R/2$ from the center of the large sphere. Point P is at a distance $2R$ from the center of the large sphere and on the joint line of the two centers. The gravitational field strength g at point P is ________.

一均勻球，原來質量爲 M，半徑爲 R。現在其離球心 $R/2$ 處挖一半徑爲 $R/2$ 的空穴，求在球心與空穴中心連線上離球心 $2R$ 處的引力場強度。

(a) $g = \dfrac{GM}{4R^2}$ (b) $g = \dfrac{5GM}{8R^2}$ (c) $g = \dfrac{3GM}{16R^2}$

(d) $g = \dfrac{GM}{2R^2}$ (e) $g = \dfrac{7GM}{36R^2}$

MC16



A small block slides down from rest at point P_1 on the surface of a smooth circular cylinder, as shown. At P_2 the particle falls off the cylinder. The equation relating the angles θ_1 and θ_2 is given by__________.

一小塊由靜止從光滑圓柱面上 P_1 點滑落，到 P_2 點離開柱面。兩角度 θ_1 、 θ_2 的關係須爲________。

(a) $\sin\theta_1 = \dfrac{3}{2}\sin\theta_2$ (b) $\sin\theta_1 = \dfrac{2}{3}\cos\theta_2$ (c) $\sin\theta_1 = \cos\theta_2$

(d) $\cos\theta_1 = \sin\theta_2$ (e) $\cos\theta_1 = \dfrac{3}{2}\sin\theta_2$

MC17

50.0 g of ice at – 40 °C is mixed with 11.0 g of steam at 120 °C. Neglect any heat exchange with the surroundings. What is the final temperature of the mixture? (Specific heats of ice, water, and steam are 0.50, 1.00, and 0.481 cal (g °C)$^{-1}$, respectively. The heat of fusion of ice is 79.8 cal/g. The heat of vaporization of steam is 540 cal/g.)

將 50.0 g 在 – 40 °C 的冰與 11.0 g 在 120 °C 的水蒸汽混合。不計與外界的熱交換。求最終溫度。（冰、水、水蒸汽的比熱分別爲 0.50、1.00、0.481 cal (g °C)$^{-1}$。冰的熔化熱爲 79.8 cal/g。水的蒸發熱爲 540 cal/g。）

(a) 35.3°C (b) 30.3°C (c) 25.3°C (d) 20.3°C

(e) 15.3°C

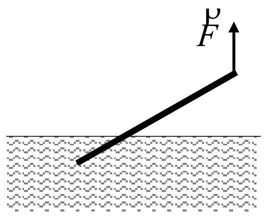
MC18

Electrons accelerated from rest by a voltage V enter a magnetic field of strength B which is perpendicular to the electron velocity. The trajectory of the electrons in the magnetic field is a circle of radius R. The electron charge to mass ratio e/m_e is then _________.

電子由靜止經過電壓 V 加速後進入強度爲 B 的磁場區，磁場方向與電子速度垂直。電子在磁場內的軌跡是半徑爲 R 的圓。由此得電子的電荷與質量之比 e/m_e 爲_____。

(a) $V/(BR)^2$ (b) $2V/(BR)^2$ (c) $4V/(BR)^2$ (d) $8V/(BR)^2$
(e) $16V/(BR)^2$

MC19

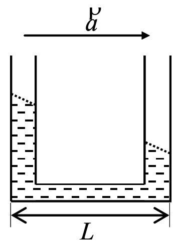


A thin uniform rod is partly immersed in water, while being lifted by a string fastened to one of its ends, as shown. If the density of the rod is 3/4 of that of water, what is the fraction of the length of the rod that is above the water when in equilibrium?

如圖。一均勻細杆，比重爲 3/4，一端吊在細繩上，部分在水裏。平衡時細杆有多少部分在水面上？

(a) 0.134 (b) 0.203 (c) 0.5 (d) 0.75 (e) 0.866

MC20



A U tube which is partially filled with liquid is in a vehicle under horizontal accelarating motion, as shown. Find the maximum difference in height of the liquid surface over the lateral distance of L.

如圖。一 U 型管，部分充有液體，在一作水平加速的車裏。求相距 L 的兩點液面的最大高度差。

(a) $\dfrac{g}{a}L$ (b) $\dfrac{a}{\sqrt{a^2+g^2}}L$ (c) $\dfrac{a}{g}L$ (d) $\sqrt{\dfrac{a}{g}}L$ (e) $\dfrac{g}{\sqrt{a^2+g^2}}L$

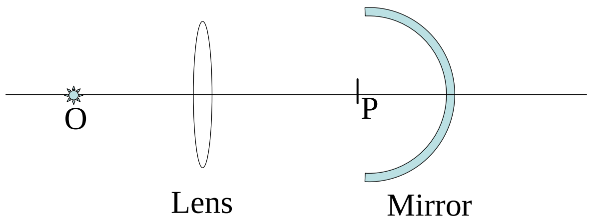
Open Problems 開放題
Total 6 problems 共 6 題

Q1 (5 points)
Describe, in less than one page, a way to determine the mass of a small object (~ kg) in weightless condition.

題 1（5 分）
以不多於一頁的篇幅，描述一種可在失重情況下確定一小物體質量（約 1kg）的方法。

Q2 (10 points)
As shown (not to actual scale), a thin convex lens with focus length 10 cm and a concave spherical mirror of radius 20 cm are placed on the same optical axis. The distance between the center of the sphere at point-P and the lens is 20 cm. A point light source O is placed on the optical axis and at 20 cm from the lens.

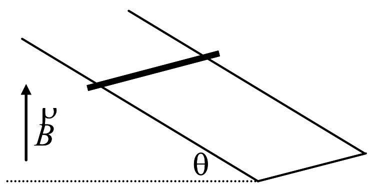
(a) Find the position of the final image of the light source.
(b) Assume the brightness of the final image in (a) is unity. The lens is then cut into two equal half disks, and the upper half is lifted by 1.0 mm from the original axis. Find the final image(s) and their brightness of the source.



題 2（10 分）
如上圖所示，一焦距爲 10 cm 的薄凸透鏡和一半徑爲 20 cm 的凹球面反射鏡共軸。P 點球心離透鏡 20 cm。一點光源 O 位於軸上離透鏡 20 cm 處。

(a) 求 O 的最終像位置。
(b) 設(a)中最終像的亮度爲 1。若將透鏡切成上下兩個半圓，上半部分提升到離原軸 1.0 mm 處，求 O 的最終像的位置和亮度。

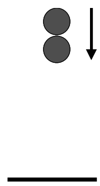
Q3 (10 points)
A conductor rod of length L, resistance R, and mass m is placed on an inclined rectangular frame made of perfect conductor in a uniform magnetic field B pointing upwards. The frame plane is at an angle θ to the horizon, as shown. Find the terminal velocity of the metal rod if
(a) there is no friction between the rod and the frame; and
(b) the friction coefficient between the rod and the frame is μ.



題 3（10 分）
如圖所示，一個理想導電金屬長方形框架以傾角 θ 放在方向向上的勻磁場 B 中，一長度爲 L, 質量爲 m, 電阻爲 R 的導體棒從框架上滑下。
(a) 求當棒和框架間無摩擦時棒的最終速度；
(b) 求當棒和框架間的摩擦係數爲 μ 時棒的最終速度。

Q4 (10 points)

Two small and hard spheres, one right on top of the other and almost in touch, are left to fall from a height H_0. The lower sphere of mass M collides with the ground, and almost instantaneously it collides with the upper sphere of mass $0.1M$. Both collisions are elastic. Find the maximum height the upper sphere can reach.



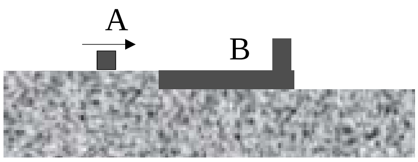
題 4（10 分）

兩個硬彈性小球，兩球連心線與地面垂直，兩球幾乎接觸，從高度 H_0 自由落下。設下球（質量爲 M）與地的碰撞以及緊接著它與上球（質量爲 $0.1M$）的碰撞都是瞬間的彈性碰撞，求上球回升的最大高度。

Q5 (12 points)

As shown, a block (Object-A) of mass m_A moves on a frictionless plane at initial speed v_0, and lands onto a cart (Object-B in the figure, mass m_B, length L, initially at rest) smoothly. Ignore the size of the block. The friction coefficient between A and B is μ and the cart is on a frictionless plane. The block collides elastically on the fixed wall at the end of the cart and eventually falls off the cart.

(a) Find the minimum value of v_0 such that the block can indeed fall off the cart.
(b) Suppose the initial speed is larger than the minimum value in (a), find the kinetic energy loss of the system (block + cart).
(c) Same condition as (b), find the time the block spends on the cart.
(d) If the initial speed is less than the minimum in (a), find the time the block spends on the cart.



題 5（12 分）

如圖，一質量爲 m_A 的小方塊以初始速度 v_0 在光滑平面上滑行，直到滑上一長度爲 L 質量爲 m_B 的靜止小車。方塊與小車之間的摩擦係數爲 μ。小車可在光滑平面上滑行。方塊之後與車固定在末端的牆作彈性碰撞，最終從車上掉下。

(a) 如方塊確能從車上掉下，初始速度 v_0 最小是多少？
(b) 如方塊的初始速度大於(a)中的最小值，求方塊在小車上所花的時間。
(c) 在(b)的過程中系統損失的動能爲多少？
(d) 如初始速度 v_0 小於(a)中的最小值，方塊在小車上所花的時間是多少？

Q6 (13 points)

This is an experiment that demonstrates the ‘matter wave’, i. e., particles such as electrons move like light wave propagation. A beam of electrons (ignore interactions between the electrons) can be viewed as a plane wave with wavelength $\lambda = \frac{h}{p}$, where h is a fundamental constant called the Planck Constant, and p is the momentum of an electron. The intensity of the wave is proportional to the electrons density. The electrons ‘matter wave’ is used to replace the light waves in a typical setting of Young’s experiment (for light waves), with two narrow slits separated by a distance d, and a large screen at a distance D (>> d) from the slits. A broad beam of electrons, accelerated by a voltage U from rest, is incident perpendicularly onto the slits plane. You can use the appropriate constants in page-2 without substituting the actual values in your answers.

(a) Find the electron density distribution on the screen, assuming that its maximum value is unity and the screen can somehow remain neutral.

(b) Add one more slit at the middle position between the two original ones. Find the electron density distribution on the screen, and its maximum value.

題 6（**13** 分）

這是一個演示‘物質波’的實驗。電子之類的微觀粒子的運動具有明顯的波動特性。一束電子如不考慮之間的相互作用可看作是一波長爲 $\lambda = \frac{h}{p}$ 的平面‘物質波’，其中 p 是單個電子的動量，h 是普朗克常數。‘物質波’的強度與電子數密度成正比。現將‘物質波’用來代替在典型的楊氏實驗裏所用的平面光波。實驗中兩窄縫的間距爲 d，到螢幕的距離爲 D (>> d)。一寬電子束由靜止經過電壓 U 加速後，准直射向窄縫平面。你可利用頁 2 上提供的常數，但不用代入其數值。

(a) 求螢幕上電子密度分佈。令密度最大值爲 1，並假設螢幕可一直保持電中性。

(b) 在兩窄縫的中間位置再開一條和原來兩條一樣的窄縫。求螢幕上電子密度分佈和分佈的最大值。

END 完

Hong Kong High School Physics Olympiad 2006
Written Examination
Answers for M.C. Questions

(1) Apply $\frac{GM}{r^2} = r\omega^2$ then plug in numbers from the table
i.e. answer is c

(2) $\begin{cases} (P_{in} - P_0)A = mg \\ P_{in}V_0 = PV \le 2PV_0 \Rightarrow M \le \frac{P_0 A - mg}{2g} \\ Mg = (P_{in} - P)A \end{cases}$

where P_{in} is the pressure inside
P is the pressure after the lift
i.e. answer is b

(3) There are three combination of a pair.
i.e. answer is b

(4) $f' = \frac{c+v'}{c-v} f$, where *f'* and *v'* is the frequency received and the velocity of the observer
i.e. answer is a

(5) $\omega = \sqrt{\frac{F}{ml}}$, which reduce to $\sqrt{\frac{g}{l}}$ as usual, when *F=mg*, which cancel out with the other m because inertial mass is equal to gravitational mass
Where $F = \sqrt{(mg)^2 + (qE)^2}$ now
i.e. answer is b

(6) Note: Far Away, so that the rays are approximately parallel
E-field double, intensity 4 times
i.e. answer is c

(7) As S_2 and S_3 is destructive at Y, the total is destructive too. (the spacing are the same)
i.e. answer is a

(8) The existence of the middle conductor is useless. The system is equivalent to a gap of 2d
i.e. answer is b

(9) The acceleration only introduce a shift of equilibrium position
i.e. answer is d

(10) By symmetry: $q_1=q_5$, $q_2=q_4$
By Kirchhoff law: $q_1=-q_2$, $q_4=-q_5$, $q_2+q_3+q_4=0$, $q_3=C_0V$, where q1, 2, 3, 4, 5 is the charge on the corresponding capacitor form left to right, and C_0 is the capacitance of individual capacitor, and V is the arbitrary voltage across AB
Express the energy stored in two different ways:

$\frac{1}{2}CV^2 = \frac{1}{2}\frac{\sum_i q_i^2}{C_0}$. $C = 2C_0$

i.e. answer is b

(11) Horizontal component along the inclines must vanish:

$$\begin{cases} T \cos(45^0 - \theta) = mg \cos 45^0 \\ T \cos(45^0 + \theta) = 3mg \cos 45^0 \end{cases} \Rightarrow 3 = \frac{\cos(45^0 + \theta)}{\cos(45^0 - \theta)}$$

Where T is the tension of the bar, θ is the angle of the bar to the horizontal
Plug in the numbers from the choices to solve the equation

i.e. answer is d

(12) If there is one more hole on the other side of the cylinder, the system is self equilibrium.
So, the system is equivalent to hold a mass filling the right hole
Considering the moment: 2*a T* = *2a/3* (80-65)

i.e. answer is c

(13) In general, V=*-5x+3y*

i.e. answer is d

(14) By Gauss law and knowing that the inside of a conductor have no E-field

i.e. answer is e

(15) Calculate the g-field of the large sphere and then subtract the contribution of the small one. It works because of the superposition principle.

i.e. answer is e

(16) $$\begin{cases} \frac{1}{2}mv^2 = mgR(\sin\theta_1 - \sin\theta_2) \\ mg\ \sin\theta_2 - N = m\frac{v^2}{R} \\ N = 0 \end{cases}$$

i.e. answer is a

(17) From the choices, we know that the mixture is in its liquid state.
Including proper treatment on the heat of fusion, the answer can be found

i.e. answer is a

(18) $$\begin{cases} \frac{1}{2}mv^2 = eV \\ evB = m\frac{v^2}{R} \end{cases} \Rightarrow v^2 = 2V\frac{e}{m} = \left(\frac{e}{m}BR\right)^2$$

i.e. answer is b

(19) $$\begin{cases} \begin{cases} f = \rho_w Vg \\ \rho_r = \frac{3}{4}\rho_w \\ \rho_r V = (1-r)M \end{cases} \Rightarrow f = \frac{4}{3}(1-r)Mg \\ f + F\ =\ Mg \\ \left(\frac{1}{2} - \frac{1-r}{2}\right)Mg = \left(1 - \frac{1-r}{2}\right)F,\ moment \end{cases} \Rightarrow \frac{4}{3}(1-r) + \frac{r}{1+r} = 1 \Rightarrow r = \frac{1}{2}$$

Where f is the buoyancy force
ρ_w is the density of water, ρ_r is the density of the rod
r is the ratio of the rod above the water surface
and V is the volume occupied under the water

i.e. answer is c

(20) By principle of equivalence, the acceleration can be treated as gravity
The water surface should perpendicular to this gravity
i.e. $\tan\theta = \frac{a}{g} = \frac{h}{L}$, where h is the height required.

i.e. answer is c

Hong Kong High School Physics Olympiad 2006
Written Examination
Answers for Open Questions

Q1 (5 points)

State the principle, the setting, and the formula to obtain the result. For example, use a spring of known force constant k to attach to the mass, and measure the vibration frequency f. Then use $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ to determine the mass m.

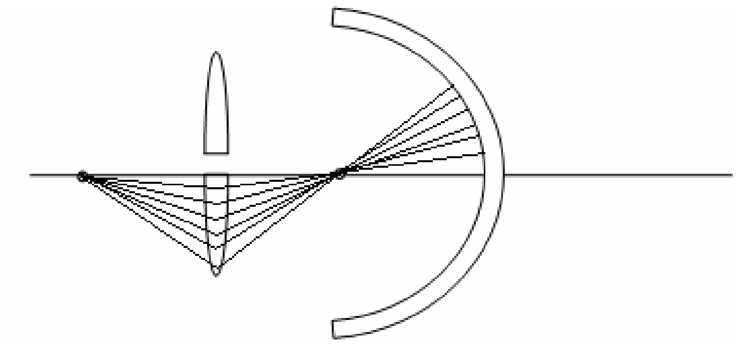
Q2 (10 points)

a) The first image due to the lens is at 20 cm from the lens, and at point-P. The second image due to the mirror is at point-P too. The final image of light source is back at the point O. (2 points)

b) Ignore the light passing through the gap since it is small comparing to the other dimension of the system. There are four possibilities:

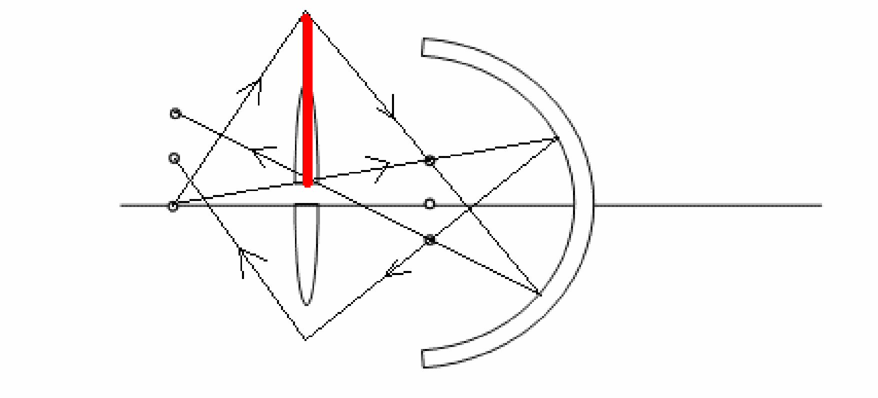
<u>Case 1:</u> The light pass through the lower half lens first. The intensity is reduced by 1/2.

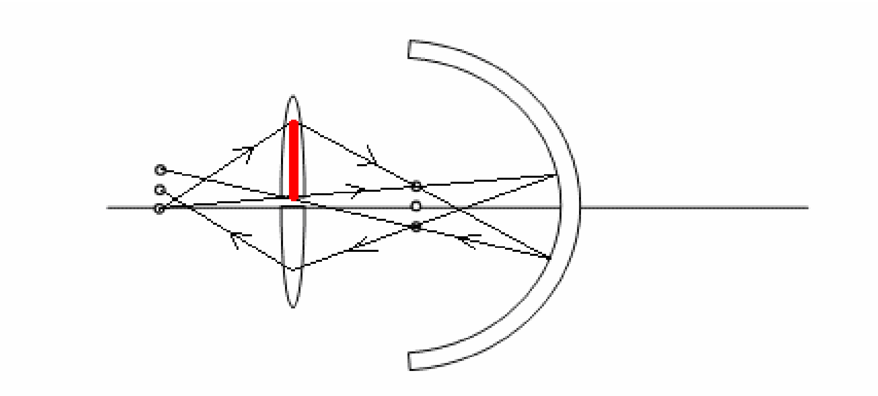
Case a: After that, by the reversibility of light, all of them will follow the same route going back to O. So, the final intensity of this spot is 1/2 of the original one. (2 points)



Case b: The light pass through the lower half lens, mirror, and then upper half lens is at 2.0mm above the source (concerning the axis of the upper lens when it travel back from the right hand side). But by the argument in (1a), after passing through the lower half lens, there will be NO light traveling trough the upper half lens. (2 points for identifying that there is no light traveling in this way)

<u>Case 2:</u> The light pass through the upper half lens first. The intensity is reduced by 1/2. After that, it can be easily found that the light can travel back via both the upper and lower half lens:





Case a: traveling back via upper half lens. By simple geometry and appropriate shifting of axis, one can find that the position of the image is 4.0mm above O.

Case b: traveling back via lower half lens. Again, one can find that the image is formed at 2.0mm above O.

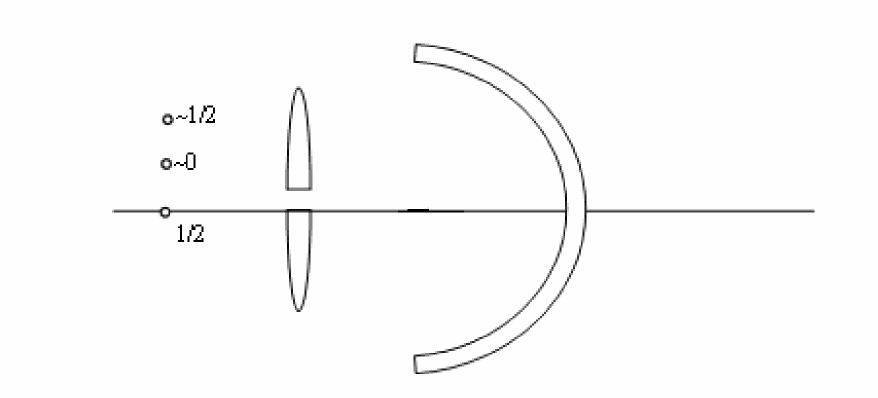
<u>To find out the intensity, neglect the effect of finite size of the lens.</u>

Method 1 (Qualitative argument):

From the above two figures, if the light coming out from the source passing through the red region, it will eventually go back via the lower half lens. As shown in the figures, the smaller the gap (which is 1.0mm in our case), the smaller the red region. In our case, the gap is very small, comparing to other physical dimension of the system. So, the red region contributes only a very small portion of the light. i.e. nearly all of them will going back via the upper half lens.

i.e. Intensity in case 2a is ~1/2.

And that in case 2b is ~0.

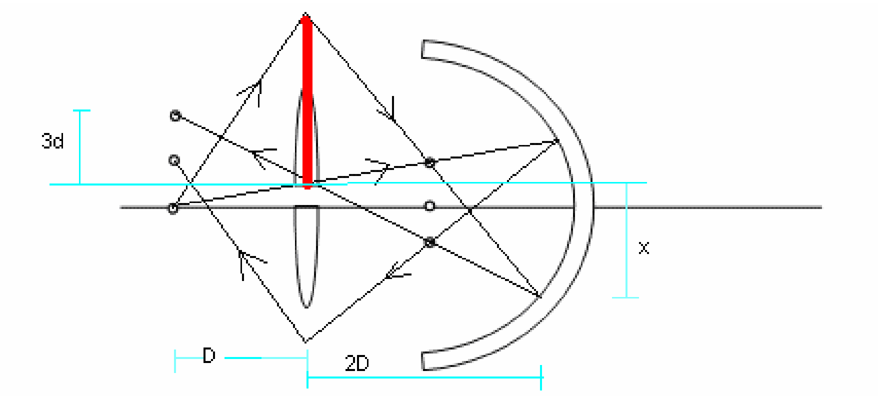


Method 2 (Quantitative argument)

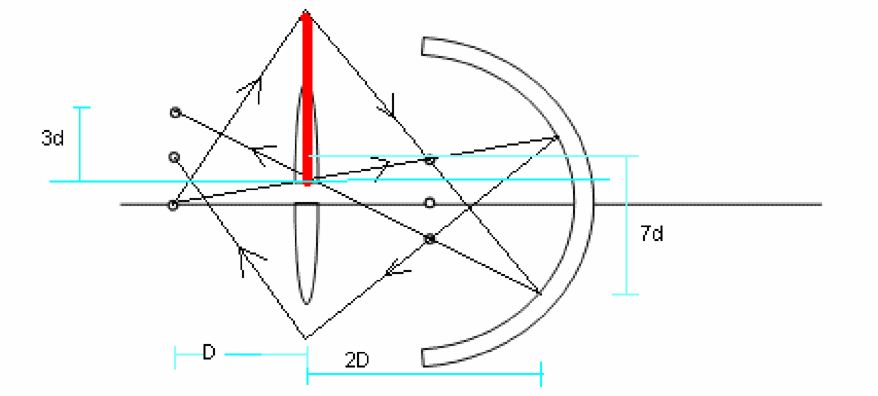
i) calculating the height of the red region first

ii) calculate the portion of light that will pass through the red region (i.e. going through the upper half lens and back via lower half lens)

answer to (i): (define d=1.0mm, D=20cm)



Small angle approximation (d<<D): x/2D = 3d/D => x = 6D



By the congruent triangle: height of red region: 7d + d = 8d

answer to (ii): the portion: $\int_0^{2\pi}\int_{\frac{\pi}{2}-\theta_1}^{\frac{\pi}{2}-\theta_2} \sin\theta d\theta d\phi$ divide by the total: $\int_0^{2\pi}\int_0^{\frac{\pi}{2}} \sin\theta d\theta d\phi$, where

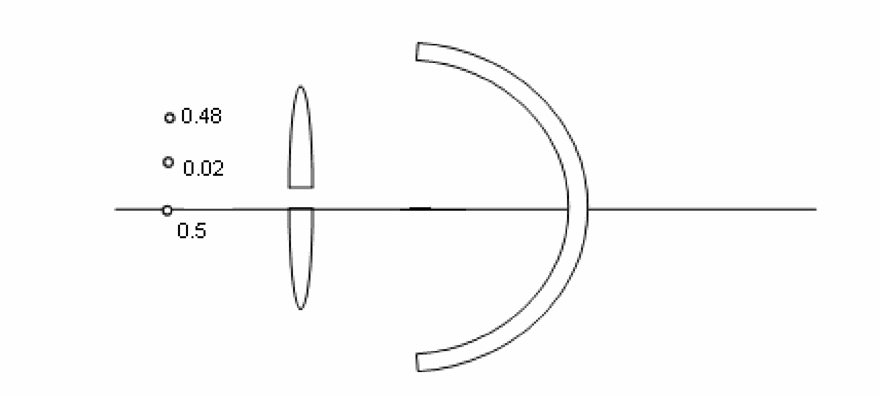
$\tan\theta_1 = \dfrac{9d}{D}$ and $\tan\theta_2 = \dfrac{d}{D}$

So, the portion of light is $\sin\theta_1 - \sin\theta_2 \approx \tan\theta_1 - \tan\theta_2 = \dfrac{8d}{D}$

Considering that the light passing through the upper half lens first is 1/2 of the original

Total intensity of the spot 2.0mm above O = 1/2 * 8d/D = 4*1.0mm/20cm = 0.02

The rest if light will focus on the spot 4.00 above O: 1-1/2-0.02 = 0.48



Q3 (10 points)

$$U = (\vec{V}\times\vec{B})\cdot\vec{L} = VLB\cos\theta \Rightarrow I = \frac{U}{R} = \frac{VLB\cos\theta}{R} \quad \text{(2 points)}$$

$$a)\, F = I\vec{L}\times\vec{B} = ILB = \frac{VL^2B^2\cos\theta}{R} \text{.................................(1)} \quad \text{(1 point)}$$

Since the rod is in equipoise, we have $F\cos\theta = mg\sin\theta$

$\therefore F = mg\tan\theta$...(2) (1 point)

combine (1) with (2), $V = \dfrac{mgR\sin\theta}{B^2L^2\cos^2\theta}$ (2 points)

b)

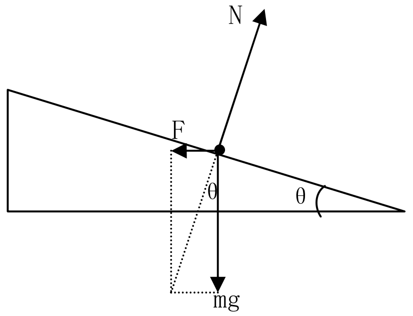
As the rod is in equipoise, we have

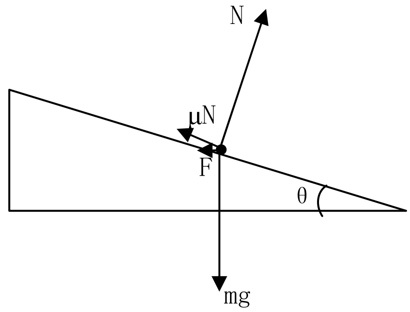
$$\mu N\sin\theta + N\cos\theta = mg \Rightarrow N = \frac{mg}{\cos\theta + \mu\sin\theta} \quad \text{(1 point)}$$

$$\mu N\cos\theta + F = N\sin\theta \quad \text{(1 point)}$$

$$\therefore F = \frac{\sin\theta - \mu\cos\theta}{\cos\theta + \mu\sin\theta}\times mg \text{..................(3)} \quad \text{(1 point)}$$

Combine(1) with(3), $V = \dfrac{mgR}{B^2L^2\cos\theta}\times\dfrac{\sin\theta - \mu\cos\theta}{\cos\theta + \mu\sin\theta}$ (2 points)





Q4 (10 points)

When a ball with mass M1 and velocity V1 collides elastically with a ball with mass M2 and velocity V2, we have

$$M_1V_1 + M_2V_2 = M_1V_1' + M_2V_2'$$

$$\frac{1}{2}M_1V_1^2 + \frac{1}{2}M_2V_2^2 = \frac{1}{2}M_1V_1'^2 + \frac{1}{2}M_2V_2'^2$$ (2 points)

Then

$$V_1' = \frac{V_1(M_1 - M_2) + 2M_2V_2}{M_1 + M_2}$$

$$V_2' = \frac{V_2(M_2 - M_1) + 2M_1V_1}{M_1 + M_2}$$ (2 points)

In this question, regard the velocity upwards to be positive, V1 = -V, V2 =V , where $V = \sqrt{2gH_0}$ is the terminal speed of the balls falling from a height H0.

$$V_1' = \frac{-V(m_1 - m_2) + 2m_2V}{m_1 + m_2} = \frac{3m_2 - m_1}{m_1 + m_2}V = \frac{3-\gamma}{1+\gamma}V$$, where $\gamma = \frac{m_1}{m_2}$ (2 points)

In this problem, γ =1/10, so $V_1' = \frac{3-0.1}{1+0.1}V$ =2.64V (2 points)

So the maximum height the top sphere can reach is 2.642 H0 = 7H0.(2 points)

Q5 (12 points)

a) Considering the critical state that the block stops at the left edge of cart, according to the law of conservation of energy, we have

$$2\mu m_A gL + \frac{1}{2}(m_A + m_B)\frac{m_A^2 V_0^2}{(m_A + m_B)^2} = \frac{1}{2}m_A V_0^2 \quad \text{(2 points)}$$

$$\Rightarrow V_0 = 2\sqrt{(1+\frac{m_A}{m_B})\mu gL} \quad \text{(1 point)}$$

b) In the coordinate system moving with the cart,

the block is decelerating with initial velocity V_0 and acceleration $-(1+\frac{m_A}{m_B})\mu g$ (2 points)

$$V_0 t + \frac{a}{2}t^2 = 2L \Rightarrow t = \frac{V_0 - \sqrt{V_0^2 - 4(1+\frac{m_A}{m_B})\mu gL}}{(1+\frac{m_A}{m_B})\mu g} \quad \text{(3 points)}$$

c) The work done by the friction is $2\mu m_A gL$. This is the amount of kinetic energy loss. (3 points)

d) Infinite (1 points)

Q6 (13 points)

The momentum of the incident electron is

$$eU = \frac{p^2}{2m_e} \quad \text{(1 point)}$$

So the wavelength of the 'matter wave' is $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2eUm_e}}$ (1 point)

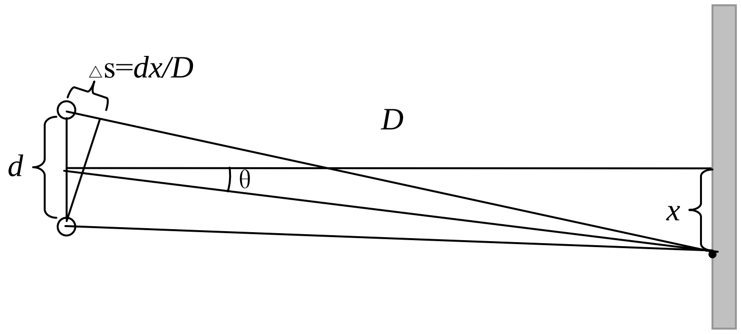
Consider a point-X which is at a distance x from the central line. The difference in distance from the two slits to point-X is given by

$$\Delta s = \frac{dx}{D} \quad \text{(2 points)}$$

$$W = W_0(1 + e^{i\frac{\Delta s}{\lambda}}) \quad \text{(2 points)}$$

$$\Rightarrow I = |W|^2 = 2W_0^2(1+\cos\frac{\Delta s}{\lambda}) = 2W_0^2(1+\cos\frac{dx}{\lambda D}) \quad \text{(1 point)}$$

$$1 = I_{\max} = 4W_0^2, \text{ so } W_0^2 = 1/4 \quad \text{(1 point)}$$



b)the distance difference changes to $\dfrac{\Delta s}{2}$ (1 point)

$$W = W_0(1 + e^{i\frac{\Delta s}{2\lambda}} + e^{-i\frac{\Delta s}{2\lambda}}) = W_0(1 + 2\cos\frac{\Delta s}{2\lambda}) \quad \text{(2 points)}$$

$$\Rightarrow I(x) = |W|^2 = W_0{}^2(1 + 2\cos\frac{\Delta s}{2\lambda})^2 = W_0{}^2(1 + 2\cos\frac{dx}{2\lambda D})^2 \quad \text{(1 point)}$$

$$I_{\max} = W_0{}^2(3)^2 = \frac{9}{4} \quad \text{(1 point)}$$