Hong Kong Physics Olympiad 2005
2005 年香港物理奧林匹克
Written Examination 筆試

Jointly Organized by
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共同舉辦

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The following symbols and constants will be used throughout the examination paper unless otherwise specified:

g – gravitational acceleration on Earth surface, 9.8 (m/s^2)
G – gravitation constant, 6.67 x 10^{-11} ($N\ m^2/kg^2$)
e – charge of an electron, –1.6 x 10^{-19} (A s)
ε_0 – electrostatic constant, 8.85 x 10^{-12} (A s)/(V m)
m_e – electron mass = 9.11 x 10^{-31} kg
c – speed of light in vacuum, 3.0 x 10^8 m/s
Radius of Earth = 6378 km
Sun-Earth distance = 1.5 x 10^{11} m
Density of water = 1.0 x 10^3 kg/m^3
Density of iron = 7.7 x 10^3 kg/m^3
Density of mercury = 13.6 x 10^3 kg/m^3
Speed of sound in air = 340 m/s

除非特別說明，本卷將使用下列符號和常數：

g – 地球表面重力加速度, 9.8 (m/s^2)
G – 重力常數, 6.67 x 10^{-11} ($N\ m^2/kg^2$)
e – 電子電荷, –1.6 x 10^{-19} (A s)
ε_0 – 靜電常數, 8.85 x 10^{-12} (A s)/(V m)
m_e – 電子質量, 9.11 x 10^{-31} kg
c – 真空光速, 3.0 x 10^8 m/s
地球半徑 = 6378 km
太陽-地球距離 = 1.5 x 10^{11} m
水的密度 = 1.0 x 10^3 kg/m^3
鐵的密度 = 7.7 x 10^3 kg/m^3
水銀的密度= 13.6 x 10^3 kg/m^3
空氣中聲速= 340 m/s

The following conditions will be applied unless otherwise specified:

1) All objects are near Earth surface and the gravity is pointing downwards.
2) Neglect air resistance.
3) All speeds are much lower than the speed of light.

除非特別說明，本卷將使用下列條件：

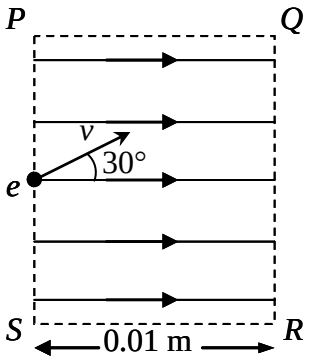
1) 所有物體都處於地球表面，重力向下；
2) 忽略空氣阻力；
3) 所有速度均遠低於光速.

Multiple choice questions (2 points each. Select one answer in each question.)
選擇題（每道題二分，每道題選擇一個答案）

[1] A car of mass m is slipping down a slope of inclination angle θ at a constant acceleration a. The static friction coefficient between the wheels and the slope is μ. What is the friction force between the wheels and the slope?
一質量爲 m 的小車從斜度爲θ的斜坡以匀加速度 a 滑落下來。其輪與坡面的靜磨擦係數爲μ。求輪與坡面的磨擦力。

(a) μmgcosθ. (b) μmg. (c) mg(sinθ - μ). (d) m(g - a).
(e) mgsinθ - ma.

[2] Refer to the figure, rectangle $PQRS$ represents the cross-section of a uniform magnetic field region of 0.20 T. An electron is projected at a speed of $v = 2.0 \times 10^6$ m/s into the region at an angle of 30° to the direction of the magnetic field. The length of the magnetic field region is 0.01 m. Find the number of revolutions made by the electron before it leaves the magnetic field region.



如圖所示。長方型區域 $PQRS$ 爲一 0.20 T 的均匀磁場的橫截面。區域長度爲 0.01 m。一電子以 $v = 2.0 \times 10^6$ m/s 的速度和與磁場成 30°的角度射入。求電子離開區域前轉的圈數。

(a) 28 (b) 16 (c) 9 (d) 6 (e) 32

[3] Two weights, both of mass m, are joined by a weightless spring of natural length l and force constant k. They are placed on a smooth surface and at rest. One weight is suddenly given an impulse and acquires an initial velocity v towards the other weight. What is the speed of the center of mass of the weights-spring system?

在光滑平面上有兩質量均爲 m 的物體，中間由一自然長度爲 l 、力常數爲 k 的輕彈簧相連。現突然給其中一物體一衝量，使它具有指向另一物體的初速度 v。求兩物體的共同質心的速度。

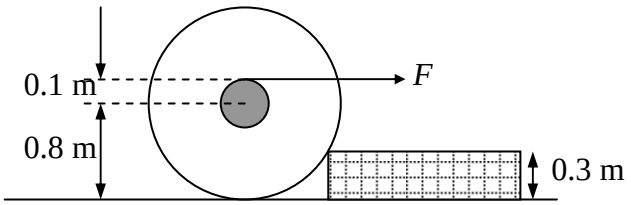
(a) 0.5v. (b) $0.5v - \sqrt{kl^2/2m}$. (c) $\sqrt{kl^2/2m} - 0.5v$.
(d) v. (e) $0.5v - \sqrt{kl^2/m}$.

[4] Following the above MC. What is the minimum distance between the two weights?
接上題。求兩物體間最小距離。

(a) $l - \frac{v}{2}\sqrt{\frac{m}{k}}$. (b) $l - v\sqrt{\frac{m}{2k}}$. (c) $l - v\sqrt{\frac{m}{k}}$. (d) $v\sqrt{\frac{m}{k}}$.
(e) $\frac{v}{2}\sqrt{\frac{m}{k}}$.

[5] As shown, a wheel of weight *W* and radius 0.8 m is placed against a 0.3 m height rectangular block fixed on the ground. The wheel has an axle of radius 0.1 m. A force *F* is applied tangentially to the axle to lift the wheel. The minimum value of *F* is___.

如圖所示。一重量爲 *W* 半徑爲 0.8 m 的輪子放在 0.3 m 高的固定方磚前。輪軸的半徑爲 0.1 m。現沿輪軸切向施力 *F* 以拉起輪子。求所需最小力。



(a) $1.05W$ (b) $0.86W$ (c) $0.69W$ (d) $0.32W$
(e) $2.45W$

[6] A helicopter is trying to land on a ship deck which is drifting south (unit vector $\vec{y}_0$) at 17m/s. A 12m/s wind is blowing from east (unit vector $\vec{x}_0$). The ship crew sees the helicopter descending at 5 m/s. Take the downwards direction as unit vector $\vec{z}_0$. What is its velocity relative to water and air?
一直升機要降落在以 17m/s 向南(單位向量 $\vec{y}_0$)飄流的船上。此時刮的是風速爲 12m/s 的東風(單位向量 $\vec{x}_0$)。船員見到直升機以 5 m/s 的速度垂直降落下來。取向下方向爲單位向量 $\vec{z}_0$。求直升機相對于水和空氣的速度。

(a) $(5\vec{y}_0 - 17\vec{z}_0)$ m/s; $(-12\vec{x}_0 + 17\vec{y}_0 + 5\vec{z}_0)$ m/s
(b) $(-12\vec{x}_0 + 17\vec{y}_0 + 5\vec{z}_0)$ m/s; $(17\vec{y}_0 + 5\vec{z}_0)$ m/s
(c) $(5\vec{z}_0)$ m/s; $(-12\vec{x}_0 + 17\vec{y}_0 + 5\vec{z}_0)$ m/s
(d) $(17\vec{y}_0 + 5\vec{z}_0)$ m/s; $(-12\vec{x}_0 + 5\vec{z}_0)$ m/s
(e) $17\vec{y}_0$ m/s; $(-12\vec{x}_0 + 5\vec{z}_0)$ m/s

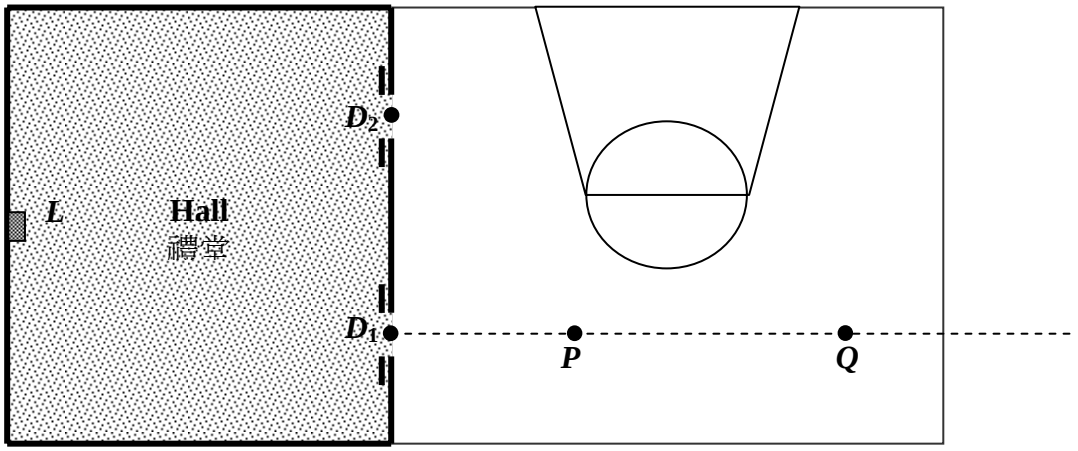
[7] Suppose the force by air to a plane is always perpendicular to its wings' surfaces. The plane is moving in a circle of radius *R* at speed *v*. The inclination angle θ of the wings should satisfy _____.
假設空氣對機翼的力總是與機翼面垂直，當飛機以速度 *v* 作半徑爲 *R* 的圓周運動時，機翼面與水平線的角度應該滿足_____。

(a) $\sin\theta = \dfrac{v^2}{Rg}$ (b) $\cos\theta = \dfrac{v^2}{Rg}$ (c) $\tan\theta = \dfrac{v^2}{Rg}$
(d) $\tan\theta = \dfrac{Rg}{v^2}$ (e) $\theta = \dfrac{Rg}{v^2}$

[8] The mass of the sun is_____.
太陽的質量爲_____。

(a) 2×10^{27} kg (b) 2×10^{28} kg (c) 2×10^{29} kg
(d) 2×10^{30} kg (e) 2×10^{31} kg

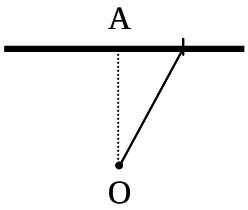
[9] A loudspeaker L is placed in the hall with two doors D_1 and D_2 open to the playground, as shown below. The distance between D_1 and D_2 is 8.5 m. The loudspeaker L is at equidistance from D_1 and D_2. Monotonic sound waves are emitted from the loudspeaker, and it is found that at point P which is 6.0 m from D_1 and at point Q the sound intensities are minimum. The line joining D_1, P and Q is perpendicular to the line joining D_1 and D_2. No other minimum intensity locations can be found between PQ and beyond Q along the PQ line. Find the frequency of the sound wave generated by the loudspeaker.



如圖，禮堂裏有一擴音器 L，離兩門 D_1 和 D_2 等距，並發出單頻聲波。兩門間距離爲 8.5 m。門外是操場。在離 D_1 6.0 m 處的 P 點和 Q 點發現聲波的強度最低。D_1PQ 連線與 D_1D_2 連線垂直。PQ 連線上 PQ 點之間和過了 Q 點後再無最弱聲波點。求聲波頻率。

(a) 17 Hz (b) 41 Hz (c) 52 Hz (d) 116 Hz
(e) 123 Hz

[10] As shown in the figure, a smooth rod is mounted horizontally on a tabletop. A 10-kg collar, which is able to slide on the rod without friction, is fastened to a spring whose other end is fixed at point-O. The nearest point of the rod to point-O is point-A, and the distance is 20 cm. The spring has a natural length of 10 cm and of negligible mass, and a spring constant of 500 N/m. The collar is released at 15 cm from point-A. Find its speed when reaching point-A.



如圖，水平桌面上有一固定的光滑杆，上套一重 10-kg 的環。一彈簧一端連著環，另一端固定在距離杆 20 cm 的 O 點。彈簧自然長度爲 10 cm，力常數 500 N/m。A 點是杆上離 O 點最近點。現將環拉到離 A 點 15 cm 處放開。求環到 A 點時的速度。

(a) 0.59 m/s (b) 0.791 m/s (c) 1.04 m/s (d) 0.88 m/s
(e) 1.24 m/s

[11] As shown, a big box of mass M is resting on a horizontal smooth floor. On the bottom of the box there is a small block of mass m. The block is given an initial speed v_0 relative to the floor, and starts to bounce back and forth between the two walls of the box. Find the final speed of the box when the block has finally come to rest in the box.

一質量爲 M 的大盒放在光滑地板上。盒底有一質量爲 m 的物體。現給該物體一初速度 v_0，使它在盒的兩壁來回碰撞。求最後物體在盒裏停下後盒的速度。

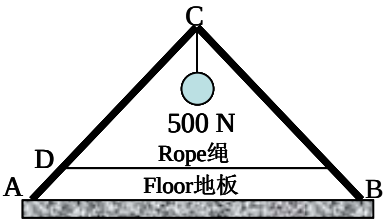
(a) 0 (b) v_0 (c) $\frac{m}{M}v_0$ (d) $\frac{M}{m+M}v_0$

(e) $\frac{m}{m+M}v_0$

[12] A jet of water from the 30cm-diameter nozzle of a fire hose can reach the maximum height of 25 meters. How large is the force from the water jet to the hose?
從直徑爲 30 cm 的消防水管射出的水柱最高可達 25 米。求水柱對水管的力。

(a) 15 kN (b) 3.46 kN (c) 346 N (d) 3.46 N
(e) 34.6 kN

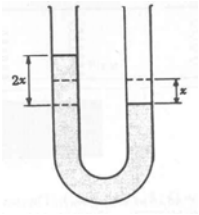
[13] As shown in the figure, AB = 3.5 m, AC = 3.0 m, AD = 0.5 m. The two rods AC and BC weight 150 N each. The floor is frictionless. Find the tension in the rope.



如圖，AB = 3.5 m, AC = 3.0 m, AD = 0.5 m。杆 AC 和 BC 各重 150 N。地板與杆間無磨擦。求繩的張力。

(a) 280 N (b) 500 N (c) 150 N
(d) 300 N (e) 180 N

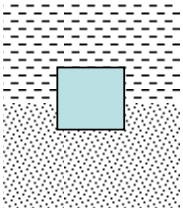
[14] 9 kg of mercury is poured into a glass U-tube with inner diameter of 1.2 cm. The mercury can flow without friction within the tube. Find the oscillation period.



一內徑爲 1.2 cm 的 U 型玻璃管裏裝有 9 kg 的水銀。設水銀可無磨擦地在管裏流動。求振盪週期。

(a) 1.2 s (b) 3.4 s (c) 5.6 s (d) 7.8 s
(e) 8.9 s

[15] A tank contains water on top of mercury. A cube of iron is sitting upright in equilibrium in the liquids. Find the fraction of its total volume in mercury.



一水缸的下部是水銀，上部是水。一立方體的鐵塊正放在液體裏。求鐵塊在水銀裏的體積與總體積之比。

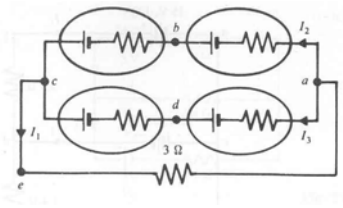
(a) 0.35 (b) 0.53 (c) 0.1 (d) 0.62 (e) 0.73

[16] The average density of Earth is _____ x 10^3 kg/m^3.

地球的平均密度爲____ x 10^3 kg/m^3。

(a) 3.1 (b) 2.2 (c) 5.5 (d) 1.1 (e) 4.1

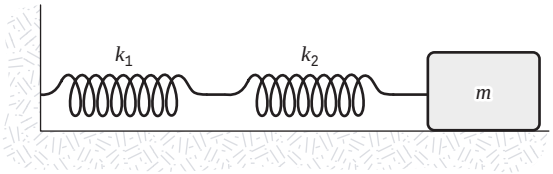
[17] Each of the four cells shown has an emf of 3.0 V and a 0.0075-Ω internal resistance. Find the current through the 3-Ω resistor.



如圖，每個電池的電動勢爲 3.0 V ，內阻爲 0.0075-Ω 。求通過 3-Ω 電阻的電流。

(a) 0.97 A (b) 0 (c) 1.95 A (d) 0.48 A
(e) 3.23 A

[18] The mass in the figure below slides on a frictionless surface. When the mass is pulled out, spring 1 is stretched a distance x_1 from its equilibrium position and spring 2 is stretched a distance x_2. The spring constants are k_1 and k_2 respectively. Find the force pulling back on the mass.



如圖，物塊可在光滑平面滑行。現將物塊拉出，使彈簧-1 拉長了 x_1 ，彈簧-2 拉長了 x_2。彈簧-1 和彈簧-2 的力常數分別爲 k_1 和 k_2。求彈簧對物塊的拉力。

(a) $-k_2x_1$. (b) $-k_2x_2$. (c) $-(k_1x_1+k_2x_2)$

(d) $-\frac{k_1+k_2}{2}(x_1+x_2)$ (e) $-\frac{k_1k_2}{k_1+k_2}(x_1+x_2)$.

[19] An empty open bottle has an inner volume of 1.31 x 10^{-4} m^3. It has a mass of 112 g when filled with air at 1 atm, and it displaces 1.63 x 10^{-4} m^3 of water when fully submerged. What fraction of the total volume of the bottle will be beneath the surface when it floats on water but without water inside the bottle?

一開口小瓶的內體積爲 1.31 x 10^{-4} m^3。裝滿 1 個大氣壓時的總質量爲 112 g。當完全浸在水裏時小瓶排開的水體積爲 1.63 x 10^{-4} m^3。現將小瓶口向上浮在水面上，瓶裏無水，求在水面下的體積部分與總體積之比。

(a) 69% (b) 18% (c) 38% (d) 100%
(e) 46%

[20] A parallel plate capacitor of capacitance C is charged to potential V by a battery. The battery is then disconnected. Which statement is correct?

一平行板電容器被加上電壓 V 後與電池斷開。以下哪個說法是正確的？

(a) There is no charge on either plate of the capacitor. 電容器板上無電荷。
(b) The capacitor can be discharged by grounding any one of its two plates. 如要電容器放電可將其中任意一板接地。
(c) Charge is distributed evenly over both the inner and outer surfaces of the plates. 電荷均勻分佈在兩板的內外表面上。
(d) The magnitude of the electric field outside the space between the plates is approximately zero. 電場在兩板間以外的空間幾乎爲零。
(e) The capacitance increases when the distance between the plates increases. 電容器的電容會因板間距離增加而增加。

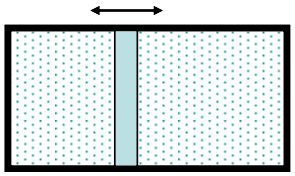
Open Problems 開放題
Total 6 problems 共 6 題

Q1 (8 points)

On a smooth and insulating ring of radius R there is a small ring of mass m and carrying charge q. The large ring is placed horizontally and in a uniform magnetic field of strength B_0 and perpendicular to the ring plane. Starting from $t = 0$, the magnetic field is changed to $B(t) = B_0 + \alpha t$. Find the force of the small ring on the big ring afterwards.

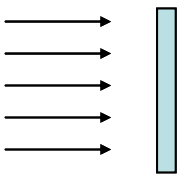
題 1（8 分）
一半徑爲 R 的光滑絕緣大圓環上有一質量爲 m 帶電爲 q 的小環。大圓環水平放置，與一強度爲 B_0 的均勻恒定磁場垂直。從時間 $t = 0$ 開始該磁場變爲 $B(t) = B_0 + \alpha t$ 。求之後小環對大圓環的力。

Q2 (8 points)
As shown in the figure, separating two sealed gas chambers is a piston of mass m and area A that can move horizontally without friction. The volume and temperature of the left chamber is V_1 and T_1, and that in the right chamber is V_2 and T_2. At equilibrium the pressure in both chambers is P. Giving the piston a small displacement Δx off balance to the right, find the maximum displacement to the left and the time needed to get there. The temperatures remain the same. (Hint: $\frac{1}{1-x} \cong 1 + x$ for $x << 1$)



題 2（8 分）
如圖所示，兩封閉氣室間有一質量爲 m 面積爲 A 的活塞可無摩擦地左右滑動。左右氣室的體積和溫度分別爲 V_1 、T_1 和 V_2 、T_2。平衡時兩邊的氣壓均爲 P。現把活塞向右推一小距離 Δx，求活塞可向左運動的最大位移和所需時間。過程中兩邊溫度不變。(提示: $\frac{1}{1-x} \cong 1 + x$ ，如果 $x << 1$)

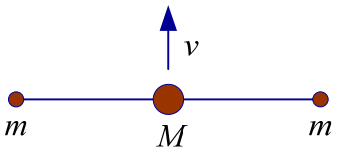
Q3 (8 points)
The shaded area in the figure is the side view of a disk shaped magnetic field region of radius R and thickness d. A parallel electron beam being accelerated by a voltage V is normally incident onto the region. Find the spatial distribution of the magnetic field (magnitude and direction) in the region such that the beam is focused to a point along the disk central axis at a distance L ($>> R$) from the disk. Your answer could also include the electron mass and charge.



題 3（8 分）
圖中陰影部分爲一半徑 R 厚度 d 的碟型磁場區的側面。一平行電子束被電壓 V 加速後垂直射入磁場區。如要求電子都被聚焦在碟軸線離磁場區距離 L ($>> R$)的點上，求磁場區裏磁場的空間分佈（方向和強度）。你的答案可包含電子的電荷和質量。

Q4 (10 points)

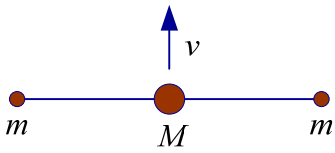
As shown, a large ball of mass M is connected on each end by a weightless thread of length l to a small ball of mass m. Initially the three balls are along the straight line on a smooth surface. The large ball is suddenly given an initial velocity v in the direction perpendicular to the line. Find



(a) The tension in the thread at the moment the large ball gets the impact;
(b) The tension in the thread at the moment the two small balls meet.

題 4（10 分）

長度爲 2 l 的輕繩，兩端各系一個質量爲 m 的小球，中央系一個質量爲 M 的大球。同一直線上的三個球均靜止於光滑的水平桌面上。現給球 M 以一個衝量，使它獲得與繩垂直的水平速度 v。試求:

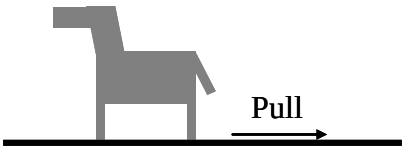


(a) 當 M 剛受到衝量時，繩中的張力。

(b) 在兩端小球發生碰撞的瞬間，繩中的張力。

Q5 (13 points)

A wooden toy horse rests on a tablecloth on a table, with its front legs 0.3 m from the cloth edge. It weighs 100 grams and its center of mass is 0.05 m from the front legs and 0.05 m above ground. The distance between the front and back legs is 0.15 m. The tablecloth is suddenly yanked horizontally with constant acceleration of 9.0 m/s^2 relative to the table. The friction coefficient between the cloth and the horse is $\mu = 0.75$. Find



(a) the acceleration of the horse relative to the table;
(b) the force on each leg of the horse by the tablecloth;
(c) the velocity and the distance the horse has traveled relative to table when the edge of the tablecloth reaches the front legs.
(d) If the height of the center of mass could be adjusted, find the value above which the horse would tip off.

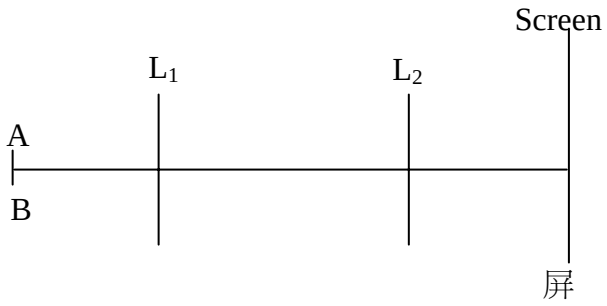
題 5（13 分）

一玩具木馬，質量爲 100 克，重心在離前腿 0.05 m，離地 0.05 m 處，前後腿距離爲 0.15 m，放在平桌面的桌布上，前腿離桌布邊 0.3 m。布與木馬間的磨擦係數爲μ = 0.75。現突然將桌布以相對於桌面 9.0 m/s^2 的加速度拉走。求

(a) 木馬相對於桌面的加速度;
(b) 桌布對木馬前後腿的力;
(c) 當木馬到達桌布邊時相對於桌面的速度和位移。
(d) 如重心高度可變，求可保持木馬不翻轉的最大重心高度。

Q6 (13 points)

As shown, L_1 and L_2 are two thin lenses sharing the same optical axis and 30 cm apart. The focus length of L_1 is 10 cm, its radius 4.0 cm, and the focus length of L_2 is 5.0 cm, and its radius 2.0 cm. AB is a bright disk object of radius 2.0 cm and 20 cm from L_1. (a) Find the position of the screen so that a clear image of the disk is formed. (b) The edge of the image is found to be dimmer than the center. Why? (c) In order make the image uniformly bright on the screen, a third lens is added. Find the position, focus length, and radius of the lens.



題 6（13 分）

如圖所示，L_1 和 L_2 是兩個共軸透鏡。兩鏡距離爲 30 cm。L_1 的焦距爲 10 cm，半徑爲 4.0 cm。L_2 的焦距爲 5.0 cm，半徑爲 2.0 cm。AB 爲一半徑爲 2.0 cm 的明亮圓碟，與透鏡共軸，距離 L_1 爲 20 cm。屏上有一圓碟的像。（a）求屏的位置。(b)爲何像的邊緣不如中心明亮？(c)爲使邊緣和中心一樣亮，需加一透鏡 L_3。求 L_3 的位置，焦距，半徑。

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Answers and suggested solutions

The Answer of Multiple-choice questions:

1. e	2. e	3. a	4. b	5. a	6. c	7. c	8. d	9. d	10. b
11. e	12. a	13. a	14. b	15. b	16. c	17. c	18. c	19. c	20. d

The Answer of Open Questions:

Q1 (8 points)

$\nabla \times \vec{E} = -\frac{\partial B}{\partial t}$ ( )

$\Rightarrow \oint E \bullet d\vec{l} = -\oint \frac{\partial B}{\partial t} ds$ ( )

$\Rightarrow E.(2\pi R) = -\alpha \pi R^2$ (2')

$E = -\frac{\alpha R}{2}$

$\because v = \frac{ft}{m} = \frac{\alpha qRt}{2m}$ (2')

$\Rightarrow F - qvB = \frac{mv^2}{R}$ (2')

$F = \frac{\alpha q^2 Rt}{4m}(2B_0 + \alpha t)$ (2')

Q2 (8 points)

$PV_1 = P_1^{'}(V_1 + A\Delta x)$ (2')

$P_1^{'} = \frac{P}{1 + (A\Delta x / V_1)} \approx P(1 - \frac{A\Delta x}{V_1})$

$PV_2 = P_2^{'}(V_2 - A\Delta x)$ (1')

$P_2^{'} = \frac{P}{1 - (A\Delta x / V_2)} \approx P(1 + \frac{A\Delta x}{V_2})$

$F = A(P_1^{'} - P_2^{'})$ (1')

$= -PA^2\left(\frac{1}{V_1} + \frac{1}{V_2}\right)\Delta x$

Which shows that the piston performs simple harmonic oscillation with $k = PA^2\left(\frac{1}{V_1} + \frac{1}{V_2}\right)$, the maximum displacement is Δx on the left side from equilibrium position. (2')

$$t = \pi\sqrt{\frac{m}{k}} = \pi\sqrt{\frac{mV_1V_2}{PA^2(V_1+V_2)}} \qquad (2)$$

Q3 (8 points)

Solution 1:

$t = \frac{d}{v}$ and $v = \sqrt{\frac{2eV}{m}}$ (2')

When the electron enters the disk, the impulse is

$m\Delta v = t(evB)$ (2')

$\Rightarrow \Delta v = \frac{evBt}{m} = \frac{eBd}{m}$

$\Rightarrow \frac{\Delta v}{v} = \frac{eBd}{mv} = \frac{r}{L}$ where $r \le R$ (3')

$\Rightarrow B = \frac{r}{dL}\sqrt{\frac{2mV}{e}}$ (1')

Solution 2:

Let r' be the radius that electron changes it direction during inside the disk,

$\frac{d}{r'} \approx \frac{R}{L}$ and $v = \sqrt{\frac{2eV}{m}}$ (2')

$evB = \frac{mv^2}{r} \quad \Rightarrow B = \frac{mv}{e}\frac{r}{dL}$ where $r \le R$ (3')

$\Rightarrow B = \frac{r}{dL}\sqrt{\frac{2mV}{e}}$ (2')

Q4: (10 points)

(a) Consider we observe the motion in the reference frame of mass M, the two small mass m will seen to be performing circular motion with initial velocity $-v$. The acceleration of M, by symmetry of the forces acting upon it, will be along $-v$ and perpendicular to the acceleration of the small masses. (1')

So, we have

$T = \frac{mv^2}{l}$ (2')

(b) Ans:

$2T_2 = Ma_M \quad \Rightarrow \quad a_M = \frac{2T_2}{M}$ (1) (1')

taking into account the initial force：

$$T_2 + ma_M = m\frac{v_x^{\ 2}}{l}, \quad (2) \qquad (1')$$

From (1) and (2) one gets $T_2 = \dfrac{Mmv_x^{\ 2}}{(M+2m)l}$. (1')

According to conservation of energy, kinetic energy of small balls in translational direction can be related as

$$2(\frac{1}{2}mv_x^2) = \frac{1}{2}Mv^2 - \frac{1}{2}(M+2m)v^2 \qquad (1')$$

$$\Rightarrow mv_x^2 = \frac{1}{2}Mv^2 - \frac{1}{2}(M+2m)\left(\frac{M}{M+2m}v\right)^2$$

$$\Rightarrow mv_x^2 = \frac{1}{2}Mv^2\left(\frac{2m}{M+2m}\right) \qquad (1')$$

Finally one gets

$$T_1 = \frac{M^2mv^2}{(M+2m)^2 l} \qquad (2')$$

Q5: (13 points)

(a) Let the acceleration of horse relative to the table be a_{ht},

$$ma_{ht} = \mu mg \qquad (1')$$

$$a_{ht} = \mu g = 0.75 \times 9.8 ms^{-2} = 7.35 ms^{-2} \qquad (1')$$

(b) Consider the net moment acting on the toy horse should be zero, we have

$$N_1 + N_2 = mg \qquad (1')$$

$$N_1 r_1 = N_2 r_2 + mg\mu h \qquad (2')$$

$$\Rightarrow (mg - N_2)r_1 = N_2 r_2 + mg\mu h$$

$$\Rightarrow N_2 = mg\frac{r_1 - \mu h}{r_1 + r_2} \qquad (1')$$

$$N_2 = (0.10kg)(9.8ms^{-2})\frac{0.05m - 0.75 \times 0.05m}{0.15m}$$

$$= 8.16 \times 10^{-2} N \qquad (1')$$

(c) Let the acceleration of horse relative to the tablecloth be a_{hc},

$$a_{hc} = a - a_{ht} = (9.0 - 7.35)ms^{-2} = 1.65 ms^{-2} \qquad (1')$$

The time required for the horse reaches the edge of tablecloth is

$$t = \sqrt{\frac{2s}{a_{hc}}} = \sqrt{\frac{2(0.3m)}{1.65ms^{-2}}} = 0.603s \qquad (1')$$

The velocity of horse relative to table at time t is

$$v_{ht} = a_{ht}t = 4.432 ms^{-1} \qquad (1')$$

The displacement on the table is

$$s' = \frac{1}{2} a_{ht} t^2 = \frac{1}{2}(7.35 ms^{-1})(0.603s)^2 = 1.336m \qquad (1')$$

(d) When the horse is fallen, $N_2 = 0$. It implies that

$$r_1 - \mu h \geq 0 \qquad (1')$$

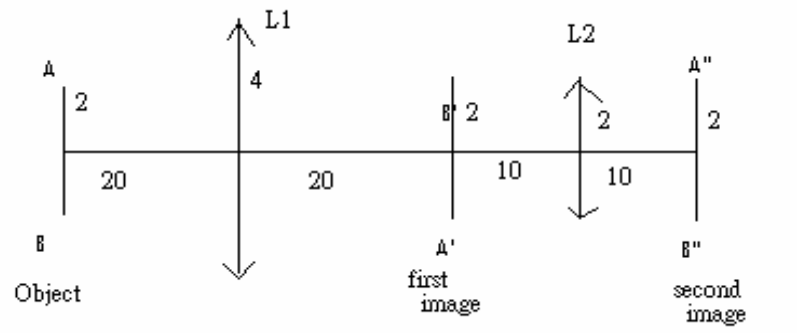
$$h \leq \frac{r_1}{\mu} = \frac{0.05m}{0.75} = 6.67 \times 10^{-2} m \qquad (1')$$

Q6 (13 points)

(a) Using the lens formula, (1')

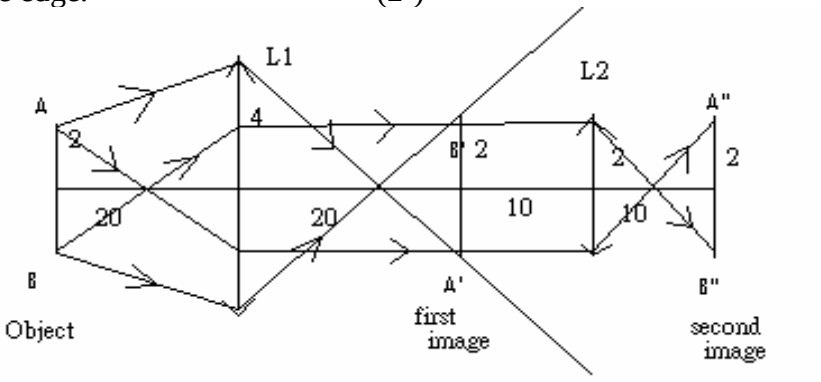
the image of AB after L_1 is at 20 cm after L_1 and 10 cm from L_2. (1')

The image after L_2 is at 10 cm from it. That is where the screen should be. (1')



(b) All the light from the center point of AB that passes through L_1 will get through L_2. (1')

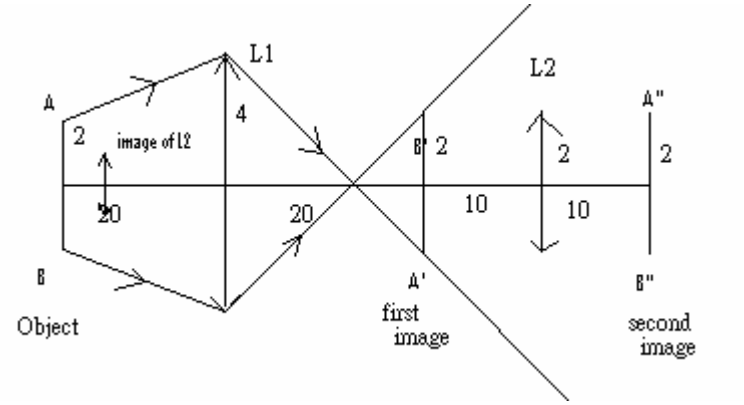
For the light from point-A which passes through the edge of L_1, using graphic method one can show that it will not pass through L_2 so it will not reach the image on the screen. Only the light through the central area of L_1 will pass L_2 and reach the screen. So the central image is brighter than the edge. (2')



Alternatively, the image of L_2 by L_1 is at 15 cm from L_1 to the left. (1')

Its size is 2 x 15/30 = 1 cm (1')

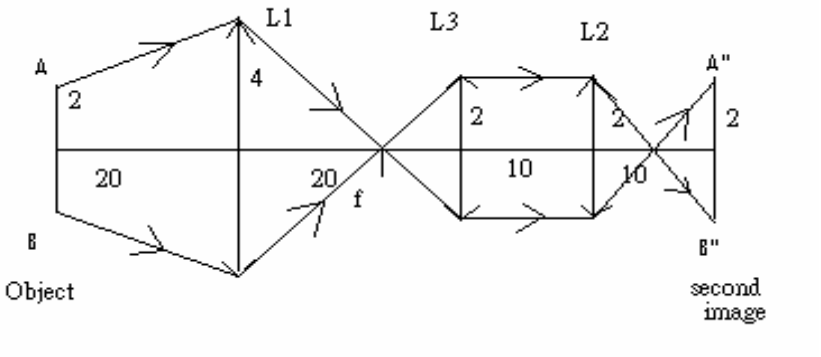
Light from the edges of object AB can reach L_1 without going through the image of L_2, and the same conclusion as above is reached.



(c) To keep the image on the screen, L_3 must be placed at the image of AB after L_1. (2')

To allow all light pass, the size of L_3 must be at least that of the AB image, which is 2 cm in radius. (2')

To determine the focus length, one may use graphic method and geometry.



This is equivalent to require the image of L_2 formed by L_3 to coincide with L_1.
The focus length is then 6.67 cm. (3')