Hong Kong Physics Olympiad 2004
2004 年香港物理奧林匹克競賽

Written Examination
筆試

Jointly Organized by

Education and Manpower Bureau
教育統籌局
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香港物理協會
The Hong Kong University of Science and Technology
香港科技大學

共同舉辦

May 30, 2004
2004 年 5 月 30 日

The following symbols will be used throughout the examination paper unless otherwise specified:

g – gravitational acceleration on Earth surface, 9.8 (m/s^2)
G – gravitation constant, 6.67×10^{-11} ($N\ m^2/kg^2$)
e – charge of an electron, -1.6×10^{-19} (A s)
ε_0 – electrostatic constant, 8.85×10^{-12} (A s)/(V m)
m_e – electron mass, 9.11×10^{-31} kg
c – speed of light in vacuum, 3.0×10^8 m/s

除非特別說明，本卷將使用下列符號：

g – 地球表面重力加速度, 9.8 (m/s^2)
G – 重力常數, 6.67×10^{-11} ($N\ m^2/kg^2$)
e – 電子電荷, -1.6×10^{-19} (A s)
ε_0 – 靜電常數, 8.85×10^{-12} (A s)/(V m)
m_e – 電子質量, 9.11×10^{-31} kg
c – 真空光速, 3.0×10^8 m/s

The following conditions will be applied unless otherwise specified:

1) All objects are near Earth surface and the gravity is pointing downwards.
2) Neglect air resistance.
3) All speeds are much lower than the speed of light.

除非特別說明，本卷將使用下列條件：

1) 所有物體都處於地球表面，重力向下；
2) 忽略空氣阻力；
3) 所有速度均遠低於光速.

Multiple choice questions (2 points each. Select one answer in each question.)
選擇題（每道題二分，每道題選擇一個答案）

MC-1
Which of the following provides the largest buoyancy to a totally submerged object?

(A) shallow sea water (B) shallow fresh water (C) deep fresh water
(D) oil (E) oil and fresh water mixture

選擇題 1
下面哪一種情況會使完全浸沒物體受到最大浮力？

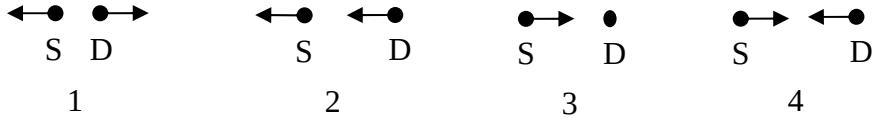
(A) 淺海水 （B）淺淡水 （C） 深淡水 （D）油 (E）油與淡水混合物

MC-2
A sinusoidal wave is traveling along a string. Any point on the string:

(A) moves in the same direction as the wave
(B) moves periodically with a different frequency from that of the wave
(C) moves periodically with the same frequency as the wave
(D) moves circularly with a different speed from that of the wave
(E) moves circularly with the same speed as the wave

選擇題 2
一正弦波在繩上傳播，繩上任意一點都_____.
（A）沿著波傳播的方向運動 （B）作與波不同頻率的周期性運動
（C）作與波相同頻率的周期性運動 （D）作與波不同速率的圓周運動
（E）作與波相同速率的圓周運動

MC-3
The diagram shows four situations in which a source of sound S and a detector D are either moving or stationary. The arrows indicate the direction of motion. The speeds are all the same. Detector 3 is stationary. Rank the situations according to the frequency detected, from lowest to highest.



(A) 1, 2, 3, 4 (B) 4, 3, 2, 1 (C) 1, 3, 4, 2
(D) 2, 1, 4, 3 (E) None of the above

選擇題 3
下圖給出運動或靜止聲源 S 與探測器 D 的四種情況。箭頭表明運動的方向。運動速率是相同的。圖 3 中探測器是靜止的。探測到的頻率由低到高的順序是

（A）1，2，3，4 （B）4，3，2，1 （C）1，3，4，2
（D）2，1，4，3 （E）以上皆不是。

MC-4
In constructing a thermometer, one *must* use a substance that:

(A) expands with rising temperature
(B) expands linearly with rising temperature
(C) will not freeze
(D) will not boil
(E) undergoes some changes when heated or cooled

選擇題 4
在組裝一個溫度計時，選用的材料**必須**是：

（A） 隨溫度上升而膨脹 （B）隨溫度上升而線性膨脹 （C）不會凝固
（D）不會沸騰 （E）加熱或冷卻時會發生變化

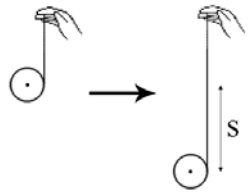
MC-5
Two identical rooms in a house are connected by an open doorway. The temperatures in the two rooms are maintained at different values. Which room contains more air?

(A) the room with higher temperature
(B) the room with lower temperature
(C) the room with higher pressure
(D) neither, because both have the same pressure
(E) neither, because both have the same volume

選擇題 5
一座房子裏有兩間由通道相連的完全相同的房間，如果兩間房間的溫度不相同，那麼哪個房間含有較多的空氣？

（A） 溫度較高的房間 （B）溫度較低的房間 （C）氣壓較高的房間
（D）都不是，因爲它們具有相同的氣壓
（E）都不是，因爲它們具有相同的體積。

MC-6
In the right figure, a light string is wound round the rim of a yo-yo of mass *m* and radius *r*. One end of the string is held by a person. When the yo-yo is released from rest, it falls and rotates at a linear acceleration of 0.8g. What is the tension in the string?

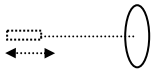


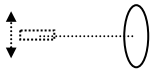
(A) 0 (B) 0.2 mg (C) 0.4 mg (D) 0.8 mg (E) mg

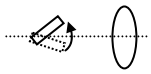
選擇題 6
右圖中一根細繩纏繞在質量爲 *m* 和半徑爲 *r* 的溜溜球的軸上。繩的一端用人手執著。當溜溜球從靜止狀態放下時，它以 0.8g 的線性加速度旋轉落下。那麼繩上的張力爲：

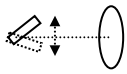
(A) 0 (B) 0.2 mg (C) 0.4 mg (D) 0.8 mg (E) mg

MC-7
A magnetic bar is in front of a coil as shown below. The line joining the center of the bar and the center of the coil (central axis) is perpendicular to the plane of the coil. Which of the following motions of the bar will **NOT** induce electric current in the coil?

(A) Translational motion back and forth. 

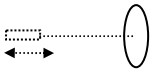
(B) Translational motion up and down. 

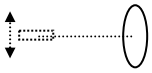
(C) The bar axis is at an angle to the central axis and spins around it. 

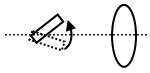
(D) The bar axis swings back and forth about the central axis within the paper plane. 

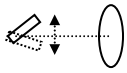
(E) The bar spins around an axis in the paper plane. 

選擇題 7
一磁棒放在如下圖所示的線圈前。磁棒中心與軸對稱的線圈中心的連線垂直於線圈的平面。下面哪種磁棒運動將**不會**產生感應電流？

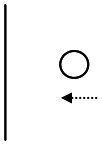
(A) 前後移動. 

(B) 上下移動. 

(C) 磁棒的軸與中心軸成一角度，並圍繞它旋轉. 

(D) 磁棒的軸在紙的平面內以中心軸上下擺動. 

(E) 磁棒以在紙的平面內的軸旋轉. 

MC-8
A coil is moving towards a straight long wire carrying a steady electric current. The wire and the motion are within the plane of the coil. The force exerted by the wire on the coil is in the direction ___. 

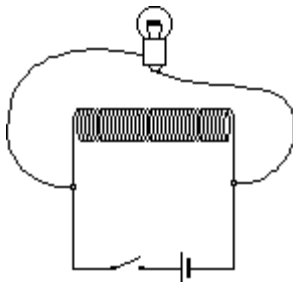
(A) away from the wire (B) towards the wire
(C) into the paper plane (D) out of the paper plane
(E) upwards

選擇題 8
線圈靠近載有穩定電流的長直導線，導線和運動方向都在線圈的平面內。那麼導線對線圈施加的力的方向是：

（A）排斥 （B）吸引 （C）朝紙內方向 （D）朝紙外方向 （E） 向上

MC-9

A battery is connected to a solenoid and a light bulb in parallel. When the switch is opened, the light bulb ___.



(A) remains off
(B) instantly goes off
(C) slowly dims out
(D) keeps burning as brightly
(E) flares up brightly, then dims and goes off

選擇題 9

一電池與線圈和燈泡並聯。當打開開關時，燈泡將___。

(A) 仍然不亮　（B）立刻熄滅　（C）慢慢熄滅
（D）保持點亮　（E）突然變亮，然後慢慢熄滅

MC-10

We have seen on TV how the astronauts were trained in weightless condition in a large airplane. To achieve weightless condition the plane should ___.

(A) Dive downwards at constant velocity
(B) Dive downwards at constant acceleration that is equal to g
(C) Accelerate in horizontal direction
(D) Move upwards at constant velocity
(E) Move upwards at constant acceleration that is equal to g

選擇題10

我們在電視上看到宇航員在一架大飛機裏處於失重狀態下訓練。爲達失重狀態，飛機應該___。

(A) 以勻速向下俯衝
(B) 以勻加速度g向下俯衝
(C) 沿水平方向加速
(D) 以勻速上升
(E) 以勻加速度g上升

MC-11

You have a manual camera with a focal length of 5cm. It is "focused" at infinity, but you want to take a picture of an object that is only 30cm away. What should you do?

(A) Move the lens out by about 1 cm
(B) Move the lens out by about 5 cm
(C) Decrease the distance between the lens and the film by about 1 cm (move the lens in)
(D) Decrease the distance between the lens and the film by about 5 cm (move the lens in more)
(E) None of the above

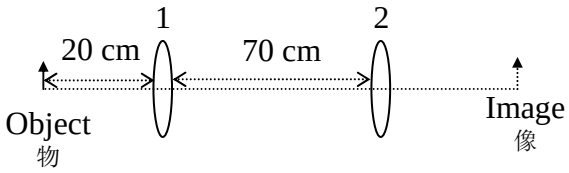
選擇題 11

你有一架焦距爲 5 釐米的手動相機。如果現在相機聚焦在無窮遠處，而你想取景的物體在 30 釐米處，那麼你應該_____

(A) 將透鏡移出約 1 釐米。
(B) 將透鏡移出約 5 釐米。
(C) 減小透鏡與膠片的位置約 1 釐米（將透鏡移入）。
(D) 減小透鏡與膠片的位置約 5 釐米（將透鏡移入很多）。
(E) 以上皆不是。

MC-12

As shown below (not drawn to the proportion), the focal lengths of both lens-1 and lens-2 are 10 cm. If a third lens identical to lens-1 is added while maintaining the image position unchanged, where should the lens be placed?



(A) Between object and lens-1, 10 cm from lens-1
(B) At the middle point between lens-1 and lens-2
(C) Between lens-1 and lens-2, 20 cm from lens-1
(D) Between lens-1 and lens-2, 20 cm from lens-2
(E) Between lens-2 and image, 10 cm from lens-2

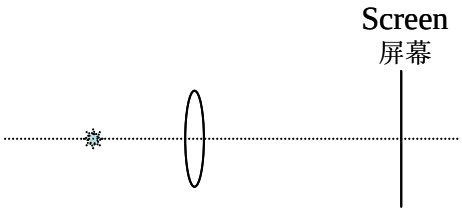
選擇題 12

如上圖所示（沒有按比例畫圖）透鏡 1 與透鏡 2 的焦距均爲 10 釐米。如果要加入第三個與透鏡 1 完全相同的透鏡，而保持成像位置不變，那麼這個透鏡應該放在__。

(A) 物體與透鏡 1 之間，且與透鏡 1 相距 10 釐米
(B) 透鏡 1 與透鏡 2 的中間位置
(C) 透鏡 1 與透鏡 2 之間，且與透鏡 1 相距 20 釐米
(D) 透鏡 1 與透鏡 2 之間，且與透鏡 2 相距 20 釐米
(E) 透鏡 2 與像之間，且與透鏡 2 相距 10 釐米

MC-13

As shown in the right figure, a point light source is placed at distance 2*f* from a lens with focus length *f*, and a screen is placed at 4*f* from the lens. The lens is then cut at the middle into two equal portions: upper half and lower half. The upper half is moved upwards by a small distance *d* comparable to the light wavelength, and the lower half is moved downwards by the same distance *d*. What is the light pattern on the screen?

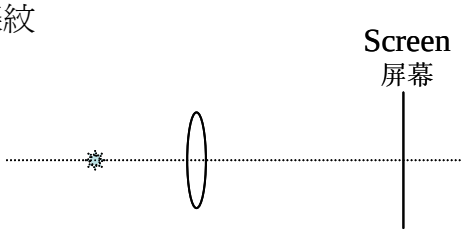


(A) Bright and dark stripes similar to the pattern seen in Young's double slit experiment
(B) Bright and dark concentric rings
(C) Two large, bright, and partly overlapping patches
(D) Two separate bright spots
(E) A large and nearly uniform light patch

選擇題 13

如右圖所示，一個點光源放在距離焦距爲 f 的透鏡 $2f$ 的位置，而螢幕放在距離透鏡 $4f$ 的位置。然後將透鏡中間切開成上下兩等份。上半個透鏡向上移動與光的波長相近的距離 d，而下半個透鏡向下移動相同的距離 d，那麼光在螢幕上的圖像是什麼？

(A) 類似在楊氏雙縫實驗中螢幕上所看到的明暗條紋
(B) 明暗相間的同心環
(C) 兩塊較大且部分重疊的明亮光斑
(D) 兩個分開的亮點
(E) 一塊較大和近似均勻的光斑。

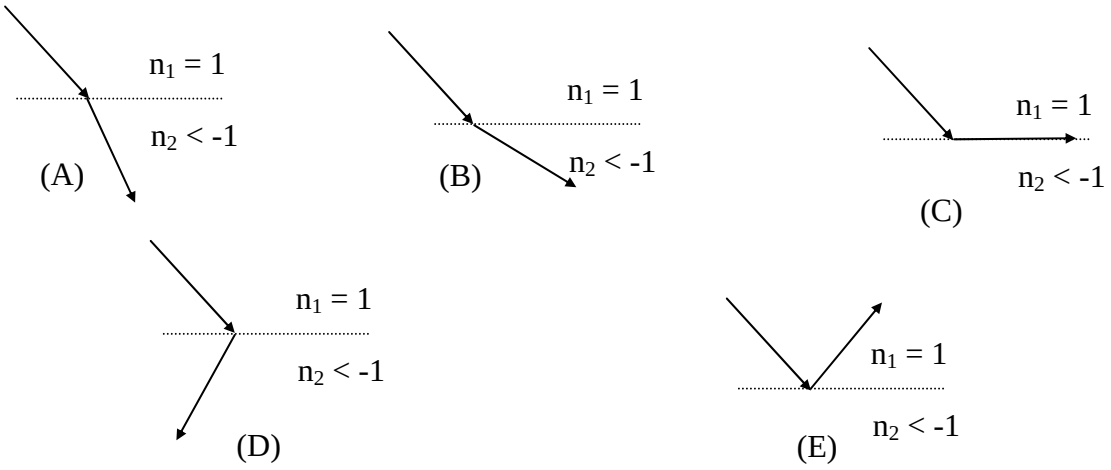


MC-14

It has been shown recently that there exists media with negative refractive index ($n_2 < -1$). If a light beam is incident on the surface of such a medium from air, which of the following describes correctly the beam entering the medium? Choice-E means that the beam cannot enter the medium. In all the others, the reflected beam is not shown, but is present.

選擇題 14

最近發現一種具有負折射率($n_2 < -1$)的介質。如果一束光從空氣入射到該介質的表面，那麼下列哪一個圖正確描述進入介質的光束? 選擇(E)表明光束不會進入介質。在所有其他圖中沒有給出反射光，但它是存在的。



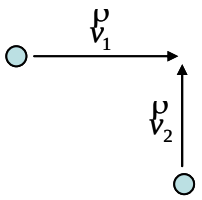
MC-15

Two balls of masses m_1, m_2 and speeds v_1 and v_2 collide at right angle. The maximum amount of kinetic energy loss due to inelastic collision is ____.

(A) $\frac{1}{2}(m_1 v_1^2 + m_2 v_2^2)$ (B) $\frac{1}{2}\frac{m_1 m_2}{m_1 + m_2}(v_1^2 + v_2^2)$

(C) $\frac{1}{2}\frac{1}{m_1 + m_2}(m_1^2 v_1^2 + m_2^2 v_2^2)$ (D) $\frac{1}{2}\frac{m_1 m_2}{m_1 + m_2}(v_1 + v_2)^2$

(E) None of the above



選擇題 15

兩個質量爲 m_1, m_2 的球分別以速率 v_1 和 v_2 垂直碰撞。那麼由於非彈性碰撞引起的最大動能損失是：

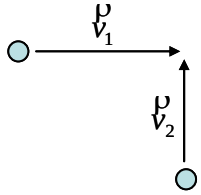
(A) $\frac{1}{2}(m_1v_1^2 + m_2v_2^2)$

(B) $\frac{1}{2}\frac{m_1m_2}{m_1+m_2}(v_1^2 + v_2^2)$

(C) $\frac{1}{2}\frac{1}{m_1+m_2}(m_1^2v_1^2 + m_2^2v_2^2)$

(D) $\frac{1}{2}\frac{m_1m_2}{m_1+m_2}(v_1 + v_2)^2$

(E) 以上皆不是



MC-16

The rebound coefficient between a tennis ball and a racket is defined as $\gamma = v_2/ v_1$, where v_1 is the incoming speed of the ball and v_2 is the speed of the ball after rebound while the racket is at rest. A tennis ball falls from height H to a racket at rest and bounces back to 0.8 H. A tennis player is using the racket to hit an incoming tennis ball traveling at 150 km/hr and the racket is moving at 100 km/hr. What is the speed of the ball after being hit? (Assume the mass of the racket >> that of the ball)

(A) 323.6 km/hr (B) 350 km/hr (C) 150 km/hr (D) 250 km/hr
(E) 234 km/hr

選擇題 16

網球與球拍的反彈係數定義爲$\gamma = v_2/ v_1$，其中 v_1 爲網球碰撞前的速率，v_2 爲網球碰撞後的速率，而球拍不動。網球從高度 H 落到靜止的球拍上，反彈高度爲 0.8 H。一個網球手用球拍以 100 km/hr 的速率撞擊以速率爲 150 km/hr 迎面而來的網球，那麼撞擊後的網球的速率是什麼？(假設球拍的質量>>網球的質量)

(A) 323.6 km/hr (B) 350 km/hr (C) 150 km/hr (D) 250 km/hr
(E) 234 km/hr

MC-17

As shown to the right, in a ^{4}He (the number on the left-up corner is the number of nucleons, i. e., protons plus neutrons, in the nucleus) cloud chamber photograph, an unknown nucleus collides with a ^{4}He nucleus, and after the collision the two nuclei travel in perpendicular directions relative to each other. If kinetic energy is lost in the collision, the unknown nucleus **must** be ___.

(A) ^{1}H (B) ^{4}He (C) ^{12}C (D) a nucleus with mass lighter than ^{4}He
(E) a nucleus with mass heavier than ^{4}He

選擇題 17

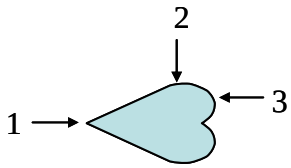
如右圖所示，在 ^{4}He（左上角的數位爲核子數，即在核子中的質子和中子數之和）雲室照片中，一個未知核子與 ^{4}He 核子碰撞，碰撞後兩個核子以彼此垂直的方向運動。如果在碰撞中動能損失了， 那麼未知的核子**一定**是：

(A) ^{1}H (B) ^{4}He (C) ^{12}C (D)質量輕於 ^{4}He 的核子
(E) 質量重於 ^{4}He 的核子

MC-18

A heart shaped conductor shown below carries net charge Q. Which of the statement about the electric field E and the surface charge density σ below is correct?

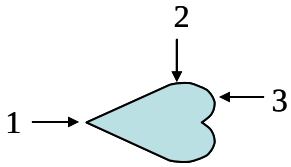
(A) E strongest and σ smallest at position-1
(B) E strongest and σ highest at position-1
(C) E weakest and σ highest at position-2
(D) E strongest and σ highest at position-3
(E) E strongest and σ highest at position-2



選擇題 18

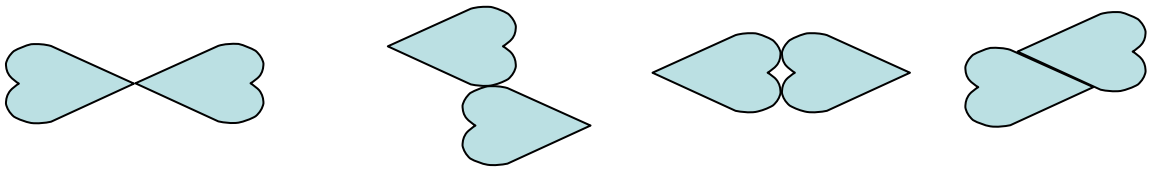
一個心形的導體如右圖所示，淨電荷爲 Q。下列哪個陳述電場 E 和表面電荷密度σ是正確的？

(A) 在位置 1 的 E 最強，σ最小
(B) 在位置 1 的 E 最強，σ最高
(C) 在位置 2 的 E 最弱，σ最高
(D) 在位置 3 的 E 最強，σ最高
(E) 在位置 2 的 E 最強，σ最高



MC-19

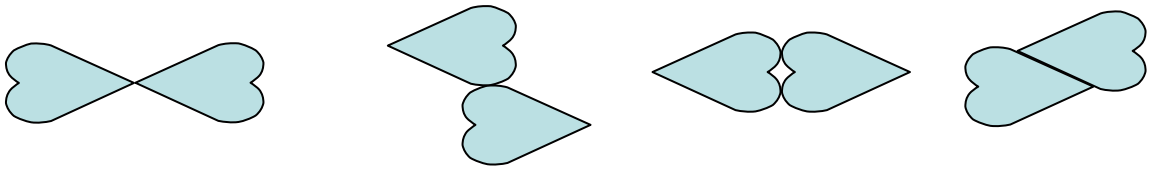
A second conductor identical to the first one, originally carrying no charge, is brought to contact with the first one which carries charge Q. In which way will the second conductor receive most amount of charge?



(A) (B) (C) (D)

(E) none of the above

選擇題 19
第二個導體與第一個完全一樣，最初並沒有帶電荷，然後與第一個帶電導體接觸。那麼按照哪種方式第二個導體可以收到最多的電荷？



(A) (B) (C) (D)

(E) 以上皆不是

MC-20
A piece of ice at 0 °C is mixed with some water also at 0 °C. If there is no heat exchange of the ice-/water mixture with the outside world, what will happen when the ice-water mixture reaches equilibrium?

(A) All ice melted (B) All water becomes ice (C) Portion of ice melted
(D) Portion of water becomes ice (E) No change in the ice/water ratio

選擇題 20
一塊處於 0 °C 的冰和一些處於 0 °C 的水混合。如果冰-水混合物與外部環境沒有熱量交換，那麼當冰-水混合物達到平衡時，會發生什麼事？

(A) 所有的冰都融化了
(B) 所有的水結冰了
(C) 部分的冰融化了
(D) 部分的水結冰了
(E) 冰水比例沒有發生改變

Hong Kong Physics Olympiad 2004
Answers and Suggested Solutions

Answers to Multiple-choice Questions:

1. A	2. C	3. A	4. E	5. B	6. B
7. C	8. A	9. E	10. B	11. A	12. C
13. A	14. D	15. B	16. A	17. D	18. B
19. E	20. E				

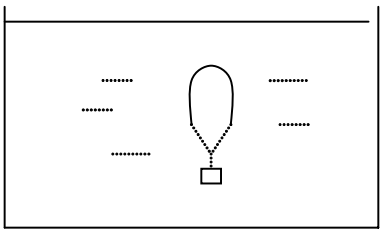
Open Questions Total 7 questions

Q-1 (5 points)
One kilogram of coarse sand is placed in a can, as shown to the right. Is the total force of the sand directly acting upon the bottom of the can larger or smaller than 9.8 N? Limit your argument to half a page.

Solution:

Due to the friction between sand grains and wall, the force on the bottom is less than 9.8 N.

Q-2 (5 points)
A rigid can with an open end is inserted into water upside down, as shown in the figure. A mass m is attached to the end of the can so that the can becomes stable when it is at a depth *h* in water. Assume air cannot escape, what happens to the motion of the can if (i) the can is moved a little bit upward from the depth *h*, (ii) downward a little bit from *h* and (iii) the can is being heated?



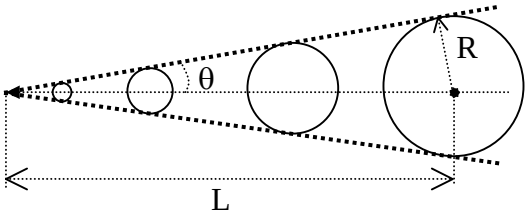
Solution:

The buoyancy of water is determined by the volume of air trapped inside the can. The volume in term is determined by PV = RT. In equilibrium at depth *h*,

$P_0V_0 = (P_0 + \rho gh)V$, and $mg = \rho gV$, where ρ is the water density, P_0 is atmosphere pressure, V_0 is volume of the can. (2 points)

(i) Decrease *h* by a little, V increases, so does buoyancy. The can will keep rising faster and faster. (1 point)
(ii) Increase *h* by a little, V decreases, so does buoyancy. The can will sink faster and faster. (1 point)
(iii) T rise causes V increase. Same motion as (i). (1 point)

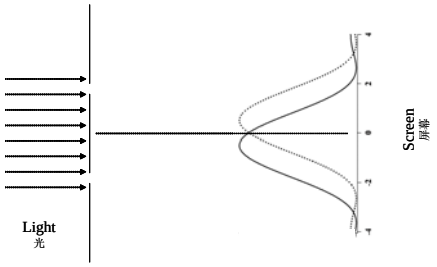
Q-3 (7 points)
The Cherenkov radiation is caused by an electron moving faster than the speed of light in a medium, such as gas. At any instance, the electron can be considered as a point source emitting spherical electromagnetic wave. Suppose the electron is not slowing down, find the shape of the wave front and its relation with the electron trajectory, in terms of the electron speed *v*, refractive index of the medium *n*, and speed of light in vacuum *c*.



(3 points)

A series of spheres form a cone-shaped wave front, as shown in the above figure. Within duration t, the radius of the wave sphere emitted at t = 0 is R = ct/n (1 point),
and the electron has traveled distance L = vt (1 point).
So sinθ = R/L = c/nv (2 points).

Q-4 (8 points)
A typical Young's double spit experiment setting is shown to the right (not drawn to the proportion). The solid curve represents the light intensity on the screen when only the lower slit is open, and the dashed curve represents the light intensity observed when only the upper slit is open. Draw a curve that qualitatively represents the light intensity on the screen when both slits are open, and briefly explain your answer (< half a page).



Solutions:

Take the coordinate x on the screen along the direction perpendicular to the slits, then at any point on the screen, the E-field from the wave through the upper slit is
$E_{upper} = A_{upper}(x)\cos(kd_1 - \omega t)$, where d_1 is the distance from point-x to the upper slit, and $A_{upper}(x)$ is given by the dashed curve. (1 point)

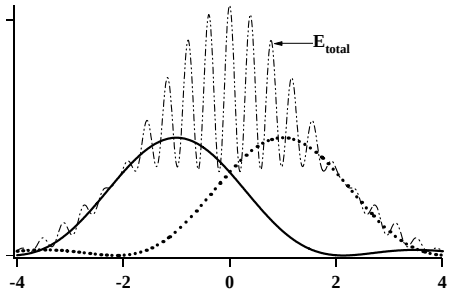
Similarly, $E_{lower} = A_{lower}(x)\cos(kd_2 - \omega t)$, where A_{lower} is given by the solid curve, and d_2 is the distance between x-point and lower slit. (1 point)

Note that $d_2 - d_1 = xd/L$, where L is the distance between slit and screen, and d is the distance between the two slits, and x is measured from the mid-point on the screen. (1 point)

When both slits are open, the total E-field at any point on the screen is then
$E = A_{upper}(x)\cos(kd_1 - \omega t) + A_{lower}(x)\cos(kd_2 - \omega t)$ (1 point)

The light intensity is $I(x) =< E^2 >= A_{upper}^2(x) + A_{lower}^2(x) + A_{upper}(x)A_{lower}(x)\cos(kxd/L)$.
(2 points)

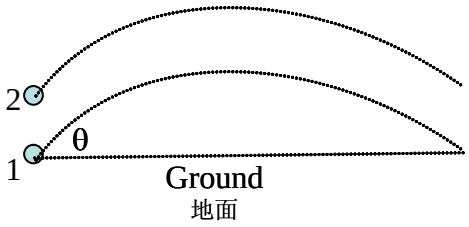
The actual light curve is show below.



(2 points)

Q-5 (10 points)

Two small solid spheres of mass *m* are thrown at the same time with the same initial velocity *v* and angle θ. Sphere-1 is thrown from the ground level while Sphere-2 is thrown at a height *h* above Sphere-1. Counting the gravitational force between the two spheres, calculate the amount of distance change δh between the two spheres at the moment Sphere-1 hits the ground. You may take the distance between the two spheres as constant *h* in your derivation for the gravitational force because $\delta h << h$. With v = 200 m/s, θ = 30 °, h = 1 m, mass of both spheres m = 1 kg, find the value of δh, and compare that to the size of an atom.



Solution:

Take the center of mass of the two balls as the reference frame, the two balls can be considered as attracting to each other via their gravity $F = Gm^2 / h^2$. (2 points)

The flying time is $t = 2v\sin\theta / g$. (1 point)

During that time the distance between the two balls has reduced by

$$\delta h = 2 \times \frac{1}{2}(F/m)t^2 \quad \text{(2 points)}$$

$$= \frac{Gm}{h^2}\left(\frac{2v\sin\theta}{g}\right)^2 \quad \text{(2 points)}$$

$$= 6.67\times10^{-11} \times 4\left(\frac{0.5\times200}{9.8}\right)^2 = 2.8\times10^{-8}(meters) \quad \text{(2 points)}$$

An atom is about $10^{-10}(meters)$ so the distance has reduced by an order of magnitude of about 200 atoms length. (1 point)

Q-6 (10 points)

In classical hydrogen atom model an electron is circling around the nucleus like Earth revolving around the sun, except that the force between the electron and the nucleus is due to the electric field. However, an electron under acceleration also emits electromagnetic waves,

and its radiation power is given by $W = \frac{e^2 a^2}{6\pi c^3 \varepsilon_0}$, where a is the acceleration of the electron.

Given the radius of the circular orbit R, find the radiation power W, and compare that with the kinetic energy E of the electron. Assuming that it roughly takes $t = E/W$ for the electron to loose all its energy and $R = 5.0 \times 10^{-11}$ m, what would be the lifetime of a hydrogen atom before it becomes a neutron?

Solution:

$a = \frac{e^2}{4\pi R^2 m \varepsilon_0}$, $W = \frac{e^2 a^2}{6\pi c^3 \varepsilon_0} = ...$ (3 points)

$a = \frac{v^2}{R} = \frac{2E}{Rm}$, (2 points)

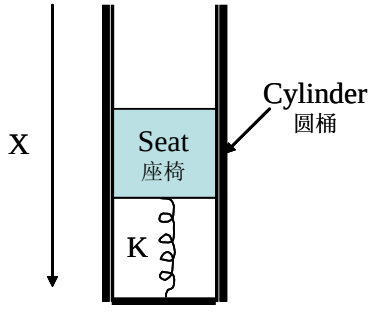
$t = \frac{E}{W} = \frac{12\pi^2 c^3 \varepsilon_0^2 m^2 R^3}{e^4}$ (3 points)

$= \frac{12 \times 3.14^2 \times 27 \times 10^{24} \times 8.85^2 \times 10^{-24} \times 9.1^2 \times 10^{-62} \times 125 \times 10^{-33}}{1.6^4 \times 10^{-76}} = 4 \times 10^{-11} s$ (2 points)

Note that because of a typo in the examination paper some students may use m = 9.1 x 10^{31} kg, and t so obtained is off by a factor of 10^{62}. Take such answer as correct one.

Q-7 (15 points)

An automatic mechanical damping system is shown to the right. The seat of mass M can move vertically inside the cylinder. The wall of the cylinder is coated with a thin layer of electro-rheological (ER) fluid. The friction force between the seat and the wall is $f = -\gamma v$, where v is the relative speed between wall and seat, and γ is the viscosity constant. The force f is always in the opposite direction of relative motion. The γ of the ER fluid can be adjusted by a voltage applied between the seat and the wall (both are metallic). At the bottom of the cylinder there is an ideal spring with force constant K and natural length d.



(a) Choose your own $x = 0$ initial position, write down the general expression for the acceleration of the seat when it is moving at velocity v at an arbitrary position x. (2 points)
(b) Find the position of the seat when everything is stationary. (2 points)
(c) An impulse then hits the cylinder from below at $t = 0$, instantly giving it an upward velocity v_0. The viscosity at this moment is γ_0. Using the cylinder as the reference frame, find the acceleration a_0 of the seat at this moment. (2 points)
(d) The active damping is now turned on and γ is controlled by the applied voltage. Find γ as a function of time such that the acceleration at t (> 0) remains to be a_0 until the seat comes to rest. (5 points)
(e) Find the distance traveled by the seat in (d). (2 points)
(f) Find the amount of energy converted to heat due to friction in (d). (2 points)

Solution:

(a) Choose x = 0 point when the spring is at its natural length, i. e., when the seat is at distance *d* from the bottom, and downward as positive. $a = \dfrac{mg - \gamma v - kx}{m}$.

(b) $mg - kx_0 = 0$, $x_0 = mg/k$.

(c) In cylinder reference frame, the seat suddenly acquires v_0 downwards at t = 0. $a_0 = \dfrac{-\gamma_0 v_0}{m}$

(d) The seat is decelerating at constant a_0, so the velocity at any time *t* is

$v(t) = v_0 - a_0 t$ (1 point)

$x(t) = x_0 + v_0 t - \dfrac{1}{2} a_0 t^2 = \dfrac{mg}{k} + v_0 t - \dfrac{1}{2} a_0 t^2$, (1 point)

Using the answer in (b), $\gamma(t) = \dfrac{mg - ma_0 - kx(t)}{v(t)}$, with a_0 given in (c), $x(t)$ and $v(t)$ given above. (3 points)

(e) Set $v(t) = 0$ in (d), we get $T = \dfrac{v_0}{a_0}$, (1 point)

and using the results in (d) we get $x(T) = \dfrac{v_0^2}{2a_0}$. (1 point)

(f) Using energy conservation. At $t = 0$ the total mechanical energy is (choose the gravitational potential energy to be zero at this point)

$E_i = \dfrac{1}{2} m v_0^2 + \dfrac{1}{2} k x_0^2$ (1 point)

The final energy is $E_f = \dfrac{1}{2} kx(T)^2 - mgx(T)$, and energy loss to friction is

$\Delta E = E_i - E_f$ =… (putting v_0, x_0, $x(T)$ in) (1 point)