

Hong Kong Physics Olympiad 2011
2011 香港物理奧林匹克

(Senior Level 高級組)

Jointly Organized by

The Hong Kong Academy for Gifted Education
香港資優教育學院

The Education Bureau of HKSAR
香港特區政府教育局

The Hong Kong Physical Society
香港物理學會

The Hong Kong University of Science and Technology
香港科技大學

共同舉辦

March 19, 2011
2011 年 3 月 19 日

Rules and Regulations 競賽規則

1. All questions are in bilingual versions. You can answer in either Chinese or English.

所有題目均為中英對照。你可選擇以中文或英文作答。

2. The multiple-choice answer sheet will be collected 1.5 hours after the start of the contest. You can start answering the open-ended questions any time after you have completed the multiple-choice questions without waiting for announcements.

選擇題的答題紙將于比賽開始後一小時三十分收回。若你在這之前已完成了選擇題，你亦可開始作答開放式題目，而無須等候任何宣佈。

3. Please follow the instructions on the multiple-choice answer sheet, and use a HB pencil to write your 8-digit Participant ID Number in the field of “I. D. No.”, and fill out the appropriate circles **fully**. After that, write your English name in the space provided and your Hong Kong ID number in the field of “Course & Section No.”

請依照選擇題答題紙的指示，用 HB 鉛筆在選擇題答題紙的 “I. D. No.” 欄上首先寫上你的 8 位數字參賽號碼，並把相應寫有數字的圓圈**完全塗黑**，然後在適當的空格填上你的英文姓名，最後於 “Course & Section No.” 欄內填上你的身分證號碼。

4. After you have made the choice in answering a multiple choice question, fill the corresponding circle on the multiple-choice answer sheet **fully** using a HB pencil.

選定選擇題的答案後，請將選擇題答題紙上相應的圓圈用 HB 鉛筆**完全塗黑**。

5. On the cover of the answer book, please write your Hong Kong ID number in the field of “Course Title”, and write your English name in the field of “Student Name” and your 8-digit Participant I. D. Number in the field of “Student Number”. You can write your answers on both sides of the sheets in the answer book.

在答題簿封面上，請於 Course Title 欄中填上你的身分證號碼；請於 Student Name 欄中填上你的英文姓名；請於 Student Number 欄中填上你的 8 位數字參賽號碼。答題簿可雙面使用。

6. The information provided in the text and in the figure of a question should be put to use together.

解題時要將文字和簡圖提供的條件一起考慮。

7. Some open problems are quite long. Read the entire problem before attempting to solve them. If you cannot solve the whole problem, try to solve some parts of it. You can even use the answers in some unsolved parts as inputs to solve the others parts of a problem.

開放題較長，最好將整題閱讀完後才著手解題。若某些部分不會做，也可把它們的答案當作已知來做其它部分。

The following symbols and constants are used throughout the examination paper unless otherwise specified:

g – gravitational acceleration on Earth surface, $9.8 \text{ (m/s}^2\text{)}$
 G – gravitational constant, $6.67 \times 10^{-11} \text{ (N m}^2\text{/kg}^2\text{)}$
 e – charge of an electron, $-1.6 \times 10^{-19} \text{ (A s)}$
 ϵ_0 – electrostatic constant, $8.85 \times 10^{-12} \text{ (A s)/(V m)}$
 m_e – electron mass = $9.11 \times 10^{-31} \text{ kg}$
 c – speed of light in vacuum, $3.0 \times 10^8 \text{ m/s}$
 Sun-Earth distance (= 1 Astronomical Unit (AU)) = $1.5 \times 10^{11} \text{ m}$
 Mass of the sun = $1.99 \times 10^{30} \text{ kg}$
 Air Density at $20 \text{ }^\circ\text{C}$ and 1 atm = 1.2 kg/m^3
 Water Density = $1.0 \times 10^3 \text{ kg/m}^3$
 Ice Density = $9.2 \times 10^2 \text{ kg/m}^3$
 Standard atmosphere pressure $p_0 = 1.0 \times 10^5 \text{ N/m}^2 = 760 \text{ mmHg}$

除非特別注明，否則本卷將使用下列符號和常數：

g – 地球表面重力加速度, $9.8 \text{ (m/s}^2\text{)}$
 G – 萬有引力常數, $6.67 \times 10^{-11} \text{ (N m}^2\text{/kg}^2\text{)}$
 e – 電子電荷, $-1.6 \times 10^{-19} \text{ (A s)}$
 ϵ_0 – 靜電常數, $8.85 \times 10^{-12} \text{ (A s)/(V m)}$
 m_e – 電子質量, $9.11 \times 10^{-31} \text{ kg}$
 c – 真空光速, $3.0 \times 10^8 \text{ m/s}$
 太陽--地球距離 (= 1 天文單位) = $1.5 \times 10^{11} \text{ m}$
 太陽質量 = $1.99 \times 10^{30} \text{ kg}$
 $20 \text{ }^\circ\text{C}$ 、一個大氣壓的空氣密度 = 1.2 kg/m^3
 水密度 = $1.0 \times 10^3 \text{ kg/m}^3$
 冰的密度 = $9.2 \times 10^2 \text{ kg/m}^3$
 標準大氣壓 $p_0 = 1.0 \times 10^5 \text{ N/m}^2 = 760 \text{ 毫米汞柱 (mmHg)}$

The following conditions will be applied to all questions unless otherwise specified:

- 1) All objects are near Earth surface and the gravity is pointing downwards.
- 2) Neglect air resistance.
- 3) All speeds are much smaller than the speed of light in vacuum.

除非特別注明，否則下列條件將適用於本卷所有問題：

- 1) 所有物體都處於地球表面，重力向下；
- 2) 忽略空氣阻力；
- 3) 所有速度均遠小於真空中的光速。

Multiple Choice Questions

(2 points each. Select one answer in each question.)

選擇題 (每題 2 分, 每題選擇一個答案。)

The MC questions with the '*' sign may require information on page-3.

帶 * 的選擇題可能需要用到第三頁上的資料。

MC1

A glass bottle falls off a table onto the floor and breaks into pieces. Which of the following statements regarding the process is **incorrect**?

一個玻璃瓶, 從桌上掉到地上摔成碎片。以下關於這一過程的陳述哪一個是**錯**的?

- (a) The potential energy of the bottle has changed.
瓶子的勢能改變了。
- (b) Some of the potential energy of the bottle has eventually been converted into the energy to break the bottle.
瓶子的一部分勢能最終轉化成將瓶子打碎所需的能量。
- (c) Some of the kinetic energy of the bottle has eventually been converted into the energy to break the bottle.
瓶子的一部分動能最終轉化成將瓶子打碎所需的能量。
- (d) The impact from the floor to the bottle is larger than the impact from the bottle to the floor.
地面對瓶子的衝量比瓶子對地面的衝量大。
- (e) The impact from the floor to the bottle breaks the bottle.
地面對瓶子的衝量打碎了瓶子。

MC2

A small weight of mass $2M$ is attached to the bottom of a hollow sphere of mass M and radius R . Half of the sphere is submerged when floating in water with mass density ρ . Find the frequency of the simple harmonic oscillation of the sphere in the vertical direction.

一個空心的球半徑為 R 、質量為 M , 下方繫了一個質量為 $2M$ 的小重物, 球浮在水面上, 有一半浸在水裏, 水的質量密度為 ρ 。求球上下浮動的簡諧振動頻率。

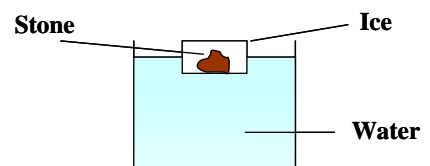
- (a) $\omega = \sqrt{2\rho g\pi R^2 / M}$
- (b) $\omega = \sqrt{\rho g\pi R^2 / (3M)}$
- (c) $\omega = \sqrt{\rho g\pi R^2 / M}$
- (d) $\omega = \sqrt{\rho g\pi R^2 / (2M)}$
- (e) $\omega = \sqrt{3\rho g\pi R^2 / M}$

MC3

A piece of ice with an embedded stone floats on the surface of water in a glass. After the ice has melted, the stone sinks to the bottom of the glass. Compared with the initial water level, what is the change of the water level in the glass, first as the ice melts, and second after the stone sinks to the bottom?

一個裝水(water)的杯子裏有一塊冰(ice)浮在水面, 冰塊裏有一塊石頭(stone)。冰融解後石頭沉到杯子底。與原來水面的高度比較, (1) 當冰還在融化時, (2) 當石塊沉到水底時, 水面的高度應該是如何改變的?

- (a) Remains the same and then rises. 不變, 然後上升。
- (b) Remains the same and then falls. 不變, 然後下降。
- (c) Remains the same all the way. 一直不變。
- (d) Rises and then falls. 先升後降。
- (e) Falls and then rises. 先降後升。



MC4*

An extra-solar planet revolving around a pulsar of 1.5 solar mass was discovered in 1992. The period of the circular orbit of the planets is 98 days. Find the distance between the pulsar and the planet in terms of astronomical units (AU).

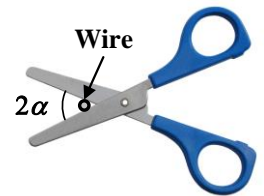
1992 年發現的太陽系以外的行星系統的中心為一個質量為太陽 1.5 倍的脈衝星。行星圓形軌道的周期為 98 天。求脈衝星與行星之間的距離（以天文單位表達）。

- (a) 0.11 (b) 0.17 (c) 0.36 (d) 0.40 (e) 0.48

MC5

Someone is using a scissors to cut a wire of circular cross section and negligible weight. The wire slides in the direction away from the hinge until the angle between the scissors blades becomes 2α . Find the friction coefficient between the blades and the wire.

用一把剪刀，去剪一條圓截面的導線。導線先會向外滑動，直到剪刀之間的角度為 2α ，求剪刀和導線之間的摩擦係數。



- (a) $\sqrt{1 - \tan \alpha}$ (b) $2 \cos \alpha$ (c) $\tan \alpha$ (d) $2 \tan \alpha$ (e) $\sqrt{2 \cos^2 2\alpha - 1}$

MC6

The kinetic energy of a particle in a simple harmonic motion is $0.5av^2$, its potential energy is $0.5bx^2$, where x is the coordinate for the position of the particle and v is its speed. The frequency of the motion is then_____.

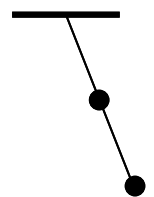
一個質點做簡諧振動，它的動能為 $0.5av^2$ 、勢能為 $0.5bx^2$ ，期中 x 是粒子的位置坐標， v 是它的速度，則簡諧振動的頻率為_____。

- (a) $\frac{1}{2\pi} \sqrt{\frac{b}{a}}$ (b) $\frac{1}{2\pi} \sqrt{\frac{a}{b}}$ (c) $\frac{1}{2\pi} \sqrt{\frac{a}{b} + \frac{b}{a}}$ (d) $\frac{1}{2\pi} \sqrt{ab}$ (e) $\frac{1}{2\pi} \sqrt{\frac{1}{ab}}$

MC7

A compound pendulum is made of a light and rigid rod of length L with one end attached to a hinge on the ceiling. A small ball of mass m is attached to the other end of the rod, and another small ball of mass $2m$ is attached onto the middle of the rod. Find the frequency of the pendulum in small amplitude simple harmonic oscillation.

一個複合單擺的輕質量剛性杆，長度為 L ，一端繫在天花板的鉸鏈上，一質量為 m 小球固定在杆的另一端點，另一個小球質量為 $2m$ 固定在杆的中點。求擺的小幅簡諧振動頻率。



- (a) $\frac{1}{2\pi} \sqrt{\frac{g}{2L}}$ (b) $\frac{1}{2\pi} \sqrt{\frac{4g}{3L}}$ (c) $\frac{1}{2\pi} \sqrt{\frac{3g}{2L}}$ (d) $\frac{1}{2\pi} \sqrt{\frac{9g}{4L}}$ (e) $\frac{1}{2\pi} \sqrt{\frac{9g}{2L}}$

MC8

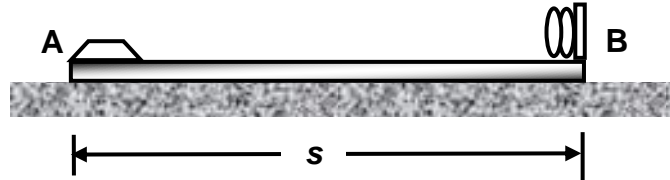
A small object is initially at the bottom of a slope inclined at an angle α with the horizontal. It is projected upward along the slope with an initial velocity, and reaches the maximum height after time t_1 . It then slides downward and returns to the initial position after time t_2 . If the coefficient of sliding friction between the object and the slope surface is μ , find the ratio t_2/t_1 .

一個傾角為 α 的斜面底部有一小物塊。現給物塊一個初速率，使它沿斜面向上滑動，經過時間 t_1 後到達最高點，之後它又滑回到底部，所用時間為 t_2 。若物體與斜面間的滑動摩擦係數為 μ ，求比例 t_2/t_1 。

- (a) $\sqrt{\frac{\sin \alpha + \mu \cos \alpha}{\sin \alpha - \mu \cos \alpha}}$ (b) $\sqrt{\frac{\sin \alpha - \mu \cos \alpha}{\sin \alpha + \mu \cos \alpha}}$ (c) $\frac{\sin \alpha - \mu \cos \alpha}{\sin \alpha + \mu \cos \alpha}$ (d) $\frac{\sin \alpha + \mu \cos \alpha}{\sin \alpha - \mu \cos \alpha}$ (e) $\sqrt{\frac{\sin \alpha}{\mu \cos \alpha}}$

MC9

As shown in the figure, AB is a board of mass $M = 4 \text{ kg}$ and length $s = 2 \text{ m}$, placed on a smooth horizontal surface. A bumper of negligible mass is fixed at end-B. A peg of mass $m = 1 \text{ kg}$ is placed at end-A. The coefficient of kinetic friction between the peg and the board is $\mu = 0.2$. With both the board initially at rest, the peg is ejected with an initial velocity of $v_0 = 10 \text{ m/s}$ along the board until it hits the bumper at end-B. After the collision, it returns to end-A without falling off the board. Find the mechanical energy loss in the process.

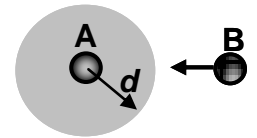


如圖所示，AB 為位于光滑水平面上的長木板，質量為 $M = 4 \text{ kg}$ ，長度為 $s = 2 \text{ m}$ 。B 端有一固定擋板。A 端放有一小滑塊，其質量 $m = 1 \text{ kg}$ 。小滑塊與木板間的動摩擦系數為 $\mu = 0.2$ 。當木板處于靜止的初始狀態時，小滑塊以初速度 $v_0 = 10 \text{ m/s}$ 沿木板射出，直到和 B 端的擋板相撞。碰撞後，小滑塊恰好回到 A 端而不脫離木板。求過程中損失的機械能。

- (a) 16 J (b) 24 J (c) 28 J (d) 32 J (e) 40 J

MC10

The elastic collision between two bodies, A and B, can be considered using the following model. A and B are free to move along a common line without friction. When their distance is greater than $d = 1 \text{ m}$, the interacting force is zero; when their distance is less than d , a constant repulsive force $F = 6 \text{ N}$ is present. The mass of body A is $m_A = 1 \text{ kg}$ and it is initially at rest; the mass of body B is $m_B = 3 \text{ kg}$ and it is approaching body A head-on with a speed $v_0 = 2 \text{ m/s}$. Find the minimum distance between A and B.

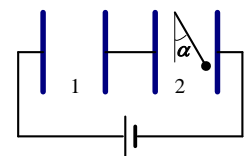


兩物體 A 和 B 的彈性碰撞過程可以如下的模型考慮。A 和 B 可沿同一直線自由運動，不用考慮摩擦力。當它們之間的距離大於 $d = 1 \text{ m}$ 時，相互作用力為零；當它們之間的距離小於 d 時，存在大小恒為 $F = 6 \text{ N}$ 的斥力。設 A 物體質量為 $m_A = 1 \text{ kg}$ ，開始時靜止在直線上某點；B 物體質量 $m_B = 3 \text{ kg}$ ，以速率 $v_0 = 2 \text{ m/s}$ 從遠處沿該直線向 A 運動。求 A、B 間的最小距離。

- (a) 0.25 m (b) 0.50 m (c) 0.75 m (d) 1 m (e) 1.25 m

MC11

Two identical parallel plate capacitors 1 and 2 are placed vertically and connected in series to a battery. In capacitor-2 there is a charged small particle attached by a thin wire to a fixed point, as shown. Ignore the effect of the charge particle on the charge distribution on the capacitor plates. At equilibrium, the angle between the wire and the vertical direction is α . Now slowly pull a plate of capacitor-1 until the distance between its two plates is doubled. After equilibrium, find the angle between the wire and the vertical direction.

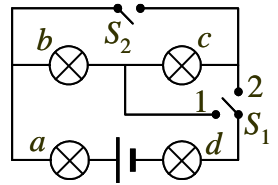


二個相同的豎直放置的平行板電容器 1 和 2 串聯後接上直流電源。電容器 2 內用細綫懸繫著一個帶電質點，平衡時懸綫與豎直方向所成的角為 α 。不考慮帶電質點對電容器 2 內電荷分布的影響。現將電容器 1 的二板間距慢慢地改變為初始時的 2 倍，電容器 2 內的帶電質點再次達到平衡。求懸綫與豎直方向所成的角。

- (a) $\tan^{-1}\left(\frac{2}{3} \tan \alpha\right)$ (b) $\tan^{-1}\left(\frac{1}{3} \tan \alpha\right)$ (c) $\tan^{-1}\left(\frac{1}{2} \tan \alpha\right)$ (d) $\tan^{-1}\left(\frac{3}{2} \tan \alpha\right)$
 (e) $\sin^{-1}\left(\frac{2}{3} \tan \alpha\right)$

MC12

Four lamps are connected in the way shown in the figure. When switch S_2 is open and switch S_1 is on position-2, Lamp-b is the brightest, and lamp-c and lamp-d are the dimmest and are of the same brightness. Now S_2 is closed and S_1 is on position-1, the sequence in brightness of the lamps is _____, with the first in the sequence being the brightest.



四盞燈按如圖所示的電路連接。當 S_2 斷開、 S_1 接 2 時，燈 b 最亮， c 和 d 最暗且亮度相同。當 S_2 閉合、 S_1 接 1 時，四盞燈按由明到暗次序排列，依次為_____。

(a) c, d, b, a (b) a, d, b, c (c) a, b, c, d (d) a, d, c, b (e) d, c, b, a

MC13

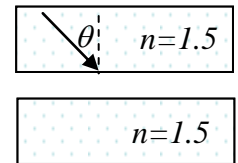
An open-end hot air balloon with fixed volume of 1000 m^3 and weighs 100 kg (including the payload but not the air inside the balloon) is floating in air. The air density outside is 1.2 kg/m^3 , and the air temperature outside is 20°C . Find the temperature of the air inside the balloon.

一個開口熱氣球，體積為 1000 m^3 ，重 100 kg (包括載重，但不包括氣球裏的空氣)，漂浮在半空。氣球外邊空氣的密度為 1.2 kg/m^3 ，氣球外邊空氣的溫度為 20°C 。求氣球裏空氣的溫度。

(a) 46.6°C (b) 20°C (c) 26.6°C (d) 146.6°C (e) 56.6°C

MC14

A beam of light with wavelength 500 nm is incident from glass (refractive index = 1.5) to an air gap at an incidence angle $\theta = 75^\circ$. To get maximum reflection, the width of the air gap should be _____.



(a) 653 nm (b) 1306 nm (c) 327 nm (d) 980 nm (e) any of the listed values

一束波長為 500 nm 的光，從玻璃(折射率等于 1.5)射到一個空氣間隙上，入射角為 $\theta = 75^\circ$ 。為達到最強反射，空氣間隙的厚度應該是_____。

(a) 653 nm (b) 1306 nm (c) 327 nm (d) 980 nm (e) 任何所列答案都對

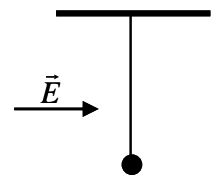
MC15

A simple pendulum is made of a small ball of mass m carrying charge q attached by a thin wire to the ceiling. Its original simple harmonic oscillation frequency is ω_0 .

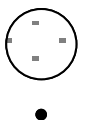
After a uniform electric field E is applied horizontally to the pendulum, its frequency becomes $2\omega_0$. Find the strength of the electric field.

一個質量為 m 的小球，由細繩繫在天花板上，成爲一個單擺。小球帶電 q ，單擺的簡諧振動頻率為 ω_0 。現在水平方向加一電場 E ，則單擺的頻率變爲 $2\omega_0$ ，求電場強度。

(a) $\sqrt{15}mg/q$ (b) $\sqrt{3}mg/q$ (c) $\sqrt{5}mg/q$ (d) mg/q (e) $2mg/q$

**MC16**

As shown, a uniform magnetic field B pointing out of the paper plane is confined in the shaded area of radius r . At a distance R from the center of the shaded area there is a point particle of mass m and carrying charge q . The magnetic field is then quickly changed to zero. Find the speed of the particle.



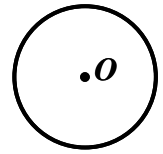
如圖，在一個半徑為 r 的陰影區有一個指出紙面的均勻磁場 B ，在離陰影區中心 R 的地方有一個質量為 m 、帶電為 q 的質點。現將磁場很快下降到零，求質點的速度。

(a) $v = \frac{qBr^2}{2mR}$ (b) 0 (c) $v = \frac{qBr^2}{mR}$ (d) $v = \frac{2qBr^2}{mR}$ (e) $v = \frac{qBr^2}{4mR}$

MC17

Shown in the figure is the top view of a long and thin wall tube of radius R carrying uniform electric current I flowing out of the paper plane. Find the magnetic field at the center of the tube.

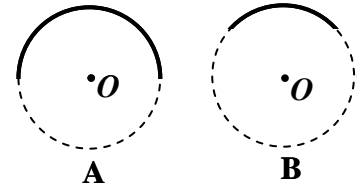
右圖所示的是一條半徑為 R 的薄壁長管的俯視圖，管壁有均勻電流 I 向紙外流出，求管中心點處的磁場。



- (a) 0 (b) $\frac{\mu_0 I}{2\pi R}$ (c) $\frac{\mu_0 I}{\pi R}$ (d) $\frac{2\mu_0 I}{\pi R}$ (e) $\frac{\mu_0 I}{4\pi R}$

MC18

Similar to MC17, structure-A is a half-circle sheet carrying uniform electric current I flowing out of the paper plane. Structure-B is a quarter-circle sheet carrying uniform electric current I flowing out of the paper plane. At the center of the circular arc point-O the total magnetic field is pointing to _____ in both cases and the field strength of structure-A is _____ than that of structure-B.

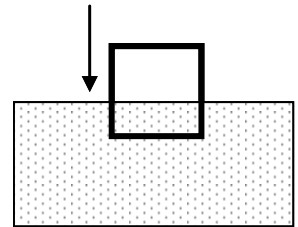


- (a) up, larger (b) left, larger (c) left, smaller (d) right, smaller (e) right, larger
- 與 MC17 相似，結構-A 為半條圓管，結構-B 為 1/4 條圓管。它們的弧度半徑一樣，流過的均勻電流也一樣，並流出紙面。則在圓弧中心 O 的磁場方向向_____，結構-A 的磁場比結構-B _____。
- (a) 上, 大 (b) 左, 大 (c) 左, 小 (d) 右, 小 (e) 右, 大

MC19

A square coil of N turns with length L , mass m , and resistance R is falling through a region of uniform magnetic field pointing out of the paper plane. The coil quickly reaches the terminal speed v , which is _____.

一個有 N 匝的正方形綫圈，邊長為 L ，質量為 m ，電阻為 R ，下降到一個均勻磁場區域，磁場指出紙面。綫圈很快達到收尾（終止）速度 v 。求速度 v 。



- (a) $v = mgR / (BL^2N)$ (b) $v = mgR / (B^2L)$ (c) $v = mgR / (B^2LN)$
 (d) $v = mgR / (BLN)$ (e) $v = mgR / (B^2L^2N)$

MC20

Similar to MC19, two square coils are falling down at terminal speed through a magnetic field region. The materials of the wires of the coils are the same, and the cross section area of the wire in coil-A is half of that of coil-B. The number of turns of coil-A is twice of that of coil-B so the two coils are of the same mass m . After the two coils fall down by a distance H ($< L$), the energy dissipated by the electric heating of coil-A and coil-B are _____, respectively.

與 MC19 類似，兩個正方形綫圈，以收尾速度下降到一個均勻磁場區域，綫圈用的材料是一樣的，綫圈 A 導綫的截面積為綫圈 B 的一半，綫圈 A 的匝數是綫圈 B 的兩倍，因此兩個綫圈具有同樣的質量 m 。兩綫圈在磁場區域下降了高度 H ($< L$)，則綫圈 A 和 B 中電熱消耗的能量分別為_____。

- (a) 0, 0 (b) mgH, mgH (c) $2mgH, mgH$ (d) $mgH, 2mgH$
 (e) $mgHR / (B^2L^2N)$, $2mgHR / (B^2L^2N)$

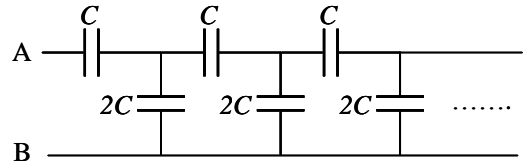
《End of MC's 選擇題完》

Open Problems 開放題

Total 5 problems 共 5 題

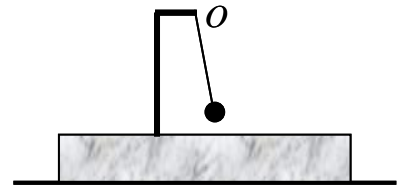
Q1 (10 points) 題 1 (10 分)

- (a) **Derive** the formulae for the total resistance of two resistors in parallel and serial connections.
推導兩個電阻串聯和并聯時總電阻的公式。
- (b) **Derive** the formulae for the total capacitance of two capacitors in parallel and serial connections.
推導兩個電容串聯和并聯時總電容的公式。
- (c) Find the capacitance between point-A and point-B of an infinitely long chain of capacitors as shown.
 圖為一個無限長電容鏈。求 A、B 點之間的電容。



Q2 (14 points) 題 2 (14 分)

A small ball of mass M_1 is attached to one end of a light but rigid rod of length L . The other end of the rod is attached to a hinge O around which the rod can rotate freely in the vertical plane. The hinge is fixed on a rectangular block with mass M_2 , which is on a smooth horizontal floor. An impulse is suddenly applied to the ball so that it acquires initial velocity v_0 to the right in the horizontal direction.



一個質量為 M_1 的小球，繫在長度為 L 的剛性細杆上，細杆的質量可以忽略，細杆的另一端繫在鉸鏈 O 點，細杆可繞 O 點在垂直平面內自由轉動，鉸鏈固定在一質量為 M_2 的長方形物塊上。物塊放在水平光滑地板上。現突然給小球一個衝量使它具有向右水平方向的初速度 v_0 。

- (a) Suppose the impulse is small enough that the ball remains near the bottom position. Find the position of the ball and the position of the block as a function of time. (7 points)
 設衝量足夠小，從而小球一直在底部附近運動，求小球和物塊之後任意時刻的位置。(7 分)
- (b) Suppose the impulse is large enough that the ball can circle around the hinge. Find the force from the rod to the ball and the force from the block to the floor when the ball is (i) at the lowest position (ii) at the same height as the hinge; and (iii) at the highest position. (7 points)
 設衝量足夠大，使小球可以繞著鉸鏈做圓周運動，求當小球在下列位置時細杆對球的力和物塊對地板的力。(i) 在最低點。(ii) 在和鉸鏈同一高度的一點。(iii) 在最高點。(7 分)

Q3 (10 points) 題 3 (10 分)

An electron has intrinsic angular momentum \vec{S} called spin, and a permanent magnetic dipole moment $\vec{M} = -\frac{ge}{2m}\vec{S}$ associated with the spin, where e is the positive electron charge, m is the electron mass, and g is a number called g-factor. An electron with its spin aligned along its initial velocity in the x -direction enters a region of uniform magnetic field in the z -direction. Show that if g is exactly 2, then the spin is always in the same direction as the velocity of the electron. (Hints: (1) The relation between torque and angular momentum is the same as the relation between force and momentum. (2) Recall the way the acceleration of uniform circular motion is derived. (3) The torque to a dipole is $\vec{\tau} = \vec{M} \times \vec{B}$.)

電子具有固有的角動量 \vec{S} ，稱為自旋，和與自旋相關的磁偶極矩 $\vec{M} = -\frac{ge}{2m}\vec{S}$ 。其中 e 為正電子電荷， m 為電子質量， g 為 g -因子。一電子初始速度和自旋都沿 X -方向，進入一均勻沿 Z -方向的磁場。證明若 g 剛好等於 2，則電子之後的速度和自旋始終在同一方向。(提示：(1)力矩與角動量的關係和力與動量的關係一樣。(2)回顧一下勻速圓周運動的加速度的推導方法。(3)磁偶極矩受到的力矩為 $\vec{\tau} = \vec{M} \times \vec{B}$ 。)

Q4* (12 points) 題 4* (12 分)

When the partial pressure of water vapor in air exceeds the saturated water vapor pressure (P_s) at a given temperature, the water vapor will condense into droplets which fall down as rain. $P_s = 55.35$ mmHg at 40°C , and $P_s = 6.50$ mmHg at 5°C . The air + vapor mixture can be considered as ideal gas and the mass of a water molecule is approximately the same as an 'air' molecule. In the humid air at sea level at 40°C the water vapor partial pressure is 90 % of P_s , and the pressure is $P_0 = 1.0$ atm.

- (a) Find the air mass density ρ_0 at the sea level. (2 points)

The humid air then rises adiabatically to an altitude where the temperature is 5°C . Ignore air pressure change due to the reduction of water vapor. For adiabatical processes of air, $PV^{7/5} = \text{Constant}$, where V is the volume, and the pressure at height H is $P = P_0(1 - 2\rho_0 gH / (7P_0))^{7/2}$, where g is the gravity acceleration near Earth surface.

- (b) How much rain can one cubic meter of the humid air at sea level generate? (7 points)

- (c) Find the altitude where the temperature is 5°C . (3 point)

當空氣中水蒸汽的分壓強超過該溫度下的飽和水蒸汽壓(P_s)時，水蒸汽將凝聚成水滴導致下雨。已知 40°C 時 $P_s = 55.35$ mmHg， 5°C 時 $P_s = 6.50$ mmHg。空氣/水蒸汽的混合物可當作是理想氣體，水分子的質量近似等于 '空氣' 分子的質量。海平面上 40°C 的潮濕空氣中，水蒸汽的分壓是 P_s 的 90 %，氣壓 $P_0 = 1$ 個大氣壓。

- (a) 求海平面處空氣的質量密度 ρ_0 。(2 分)

該潮濕空氣絕熱上升到某一高度，該處溫度為 5°C 。忽略由于水蒸汽的減少導致的氣壓改變。空氣絕熱過程狀態方程為 $PV^{7/5} = \text{常數}$ ， V 為體積，在高度 H 的壓強為 $P = P_0(1 - 2\rho_0 gH / (7P_0))^{7/2}$ 。

- (b) 一立方米海平面上的潮濕空氣能夠產生多少雨？(7 分)

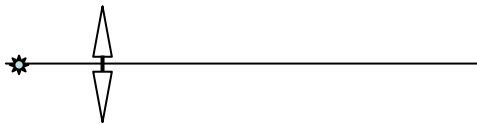
- (c) 求溫度為 5°C 處的高度。(3 分)

Q5 (14 points) 題 5 (14 分)

Refer to the figure which is not drawn to the correct scale. A positive lens of radius $R = 1.0$ cm and focal length $f = 20$ cm is cut in the middle. The upper part is lifted up by $d = 1.0$ mm and the lower part pulled down by the same distance. The gap in between is blocked by an opaque sheet. A point light source with wavelength $\lambda = 500$ nm is placed on the optical axis at $2f$ distance from the split lens. A large screen is placed at $L = 1.0$ m from the right focus point of the lens.

- (a) Find the number of interference fringes on the screen. (7 points)

- (b) The point source is pulled up by a small distance b from the optical axis. A second identical source is placed at a distance $2b$ below the first source. Find the minimum value of b to make all the fringes on the screen disappear. (7 points)



參考示意圖，一個正透鏡半徑為 $R = 1.0$ cm，焦距為 $f = 20$ cm，現將透鏡從中間割開，將上半部往上移 $d = 1.0$ mm，下半部往下移 $d = 1.0$ mm，兩個半透鏡之間的空隙由不透明物體封住。一個波長為 $\lambda = 500$ nm 的點光源，放在光軸上離透鏡的距離 $2f$ 處。一個大的屏幕放在離右焦點 $L = 1.0$ m 處。

- (a) 求屏幕上干涉條紋的數目。(7 分)

- (b) 將點光源向上移動一個小距離 b ，另一個相同的光源放在第一個光源以下距離 $2b$ 處，求能使屏幕上干涉條紋消失的 b 的最小值。(7 分)

《END 完》