Hong Kong Physics Olympiad 2011 (Junior Level)
2011 香港物理奧林匹克(初級組)
Suggested Solutions 答案及建議題解

Multiple Choice Questions

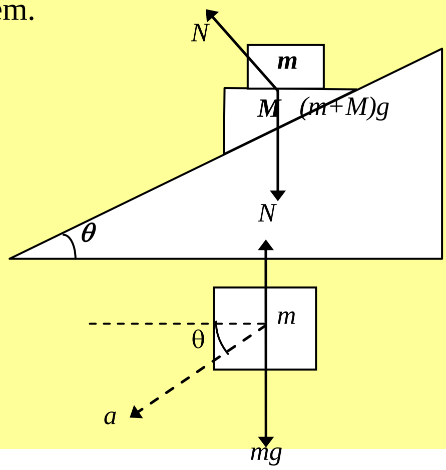
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
b	e	e	d	e	b	a	e	a	b	d	a	b	b	e	c	d	e	b	c

MC1

We may consider the wedge and the object as one system.
By using Newton's 2nd law of motion, we have

$(m + M)g\sin\theta = (m+M)a$

$\therefore\ a = g\sin\theta$ ....(1)



Consider the object:

$mg - N = ma\sin\theta$ .... (2)

sub (1) into (2), we have

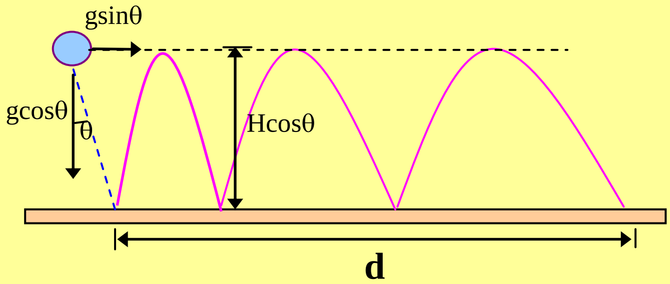
$mg - N = m(g\sin\theta)\sin\theta = mg\sin^2\theta$

$N = mg - mg\sin^2\theta = mg\cos^2\theta$

Answer: (b)

MC2

In the frame rotated by an angle θ, we can consider only the horizontal component of the gravitational force that accelerates the ball forward. In the vertical motion, the ball bounces elastically with an effective acceleration $g\cos\theta$. To find the distance between the first and the last bounce, say the fourth, we should also account for the velocity the ball acquires before the first hit:



Denote v_o as the horizontal component of the initial velocity of the ball before the first hit with respect to the inclined, t_{01} as the time between releasing the ball and the first hit and t_{14} as the time between the first to the fourth hit.

The horizontal component of the initial velocity of the ball before the first hit is given by $v_o = g\sin\theta \cdot t_{01}$ … (1)

The time t_{01}: $t_{01} = \sqrt{\dfrac{2H\cos\theta}{g\cos\theta}} = \sqrt{\dfrac{2H}{g}}$ …(2)

Since the ball makes 3 hits and each hit takes 2 times of t_{01}, therefore,

$$t_{14} = \left[2\sqrt{\frac{2H}{g}} \right] \cdot 3 = 6\sqrt{\frac{2H}{g}} \quad \ldots(3)$$

The total distance between the first and the last hit, d, is

$$d = v_0 t_{14} + \frac{(g\sin\theta)}{2} \cdot t_{14}^2 \quad \ldots.(4)$$

By substituting equation (1), (2) and (3) into (4), we have

$$d = \left[g\sin\theta \cdot \sqrt{\frac{2H}{g}} \right] \left(6\sqrt{\frac{2H}{g}} \right) + \frac{(g\sin\theta)}{2} \cdot \left[6\sqrt{\frac{2H}{g}} \right]^2$$
$$= 12H\sin\theta + 36H\sin\theta$$ **Answer: (e)**
$$= 48H\sin\theta$$

MC3*

Using Newton's law of universal gravitation and Newton's second law for circular orbits, $M \propto R^3/T^2$. Comparing with Earth's orbit, where M = 1 solar mass, R = 1 AU and T = 1 year, $R = \sqrt[3]{1.5\left(\frac{98}{365}\right)^2} = 0.48\text{ AU}$. **Answer: (e)**

MC4

Let the x axis points in the direction along the plane, and the y axis is normal to the plane. Let m be the mass of the ball. Let u be the velocity of the ball immediately before it touches the surface. Then the impulse of the impact is given by

$$I_x = mv_x - mu\cos\alpha$$
$$I_y = mv_y + mu\sin\alpha$$

Due to the presence of friction, $I_x = -\mu I_y$

Hence $mv_x - mu\cos\alpha = -\mu(mv_y + mu\sin\alpha)$

$$v_x = u(\cos\alpha - \mu\sin\alpha) - \mu v_y \qquad (1)$$

Balance of energy:

$$\frac{1}{2}mv_x^2 + \frac{1}{2}mv_y^2 = \frac{f}{2}mu^2 \text{ where } f = \frac{5}{16}$$
$$v_x^2 + v_y^2 = fu^2$$

Substituting (1),

$$[u(\cos\alpha - \mu\sin\alpha) - \mu v_y]^2 + v_y^2 = fu^2$$

we arrive at a quadratic equation in v_y:

$$(1+\mu^2)v_y^2 - 2\mu(\cos\alpha - \mu\sin\alpha)uv_y + u^2(\cos\alpha - \mu\sin\alpha)^2 - fu^2 = 0$$
$$\frac{5}{4}\left(\frac{v_y}{u}\right)^2 - \frac{1}{2}\left(\frac{v_y}{u}\right) - \frac{1}{16} = 0$$

$v_y = \dfrac{u}{2} \quad v_x = \dfrac{u}{4}$

Angle with the x axis: $\theta = \tan^{-1}\dfrac{v_y}{v_x} = \tan^{-1} 2 = 63^{\circ}$ **Answer: (d)**

MC5

Consider the motion of the center of mass of the sand. At t = 0, the center of mass starts to move downwards. Hence there is a downward acceleration. Using Newton's second law, the reaction force of the balance on the hourglass should decrease, so that it cannot completely cancel the weight of the sand, and the net force on the sand is downward. Throughout the dripping process, the center of mass of the sand is moving downwards at a constant velocity. There is no acceleration and hence the reaction force stays at the equilibrium value. At the end of the dripping process, the downward motion of the center of mass is stopped. There is a deceleration and the reaction force increases momentarily. **Answer: (e)**

MC6

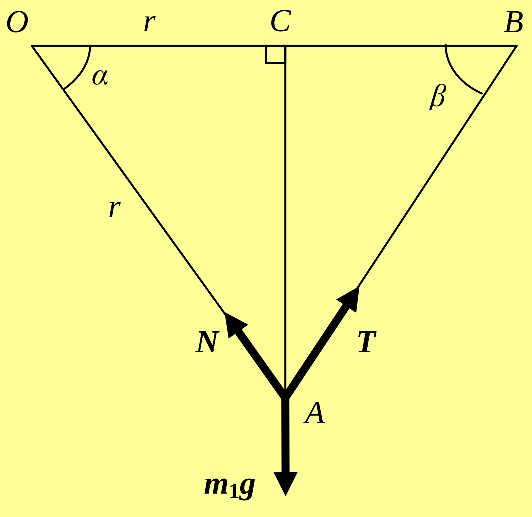
Using Archimedes' principle, volume of water displaced $V_{\text{dis}} = (m_{\text{stone}} + \rho_{\text{ice}} V_{\text{ice}}) / \rho_{\text{water}}$. Suppose a volume of ΔV_{ice} melts, then the volume of water displaced decreases by $\rho_{\text{ice}} \Delta V_{\text{ice}} / \rho_{\text{water}}$. At the same time, since mass is conserved, the volume of water increases by $\rho_{\text{ice}} \Delta V_{\text{ice}} / \rho_{\text{water}}$. Hence there is no change in the water level. However, when all ice melts and the stone sinks to the bottom, the volume of water displaced is $m_{\text{stone}} / \rho_{\text{stone}} < m_{\text{stone}} / \rho_{\text{water}}$. Hence the water level falls. **Answer: (b)**

MC7

The key is the conservation of energy. The electrical energy generated by the turbines comes from the kinetic energy of the wind, which in turn comes from the kinetic energy of the trains. Hence no energy is saved, and the air resistance experienced by the trains will increase. **Answer: (a)**

MC8

Sol. 考慮球 1 的平衡，作出受力圖如下。



$T = m_2 g$, $\beta = 90^0 - \dfrac{\alpha}{2}$.

$\Sigma F_y = 0, T\sin\beta + N\sin\alpha - m_1 g = 0$, $N\sin\alpha = m_1 g - m_2 g\sin\beta$ (i)

$\Sigma F_x = 0, T\cos\beta - N\cos\alpha = 0$, $N\cos\alpha = m_2 g\cos\beta$ (ii)

(i)÷(ii): $\dfrac{m_1}{m_2} = \dfrac{\tan\alpha}{\sqrt{2(1-\cos\alpha)}} = \sqrt{3}$. **Answer: (e)**

MC9

The force which is responsible for restoring the liquid levels in two arms of the tube is $F = -\Delta pA = -(h_1 + h_2)\rho gA$, where Δp is the pressure difference and h_1 & h_2 being the rise and fall of liquid levels in the two arms in vertical direction respectively.

Denote x as the change in length of the liquid along the tube. (note: the change in length of the liquid in both arms, either rise or fall, is the same)

Using this restoring force can enable us to have the following equation:

$F = -(x\sin\theta_1 + x\sin\theta_2)\rho gA = -\rho gA(\sin\theta_1 + \sin\theta_2)x = ma$

$$\Rightarrow a = -\frac{\rho gA(\sin\theta_1 + \sin\theta_2)}{m}\cdot x = -\omega^2 \cdot x$$

By using S.H.M. equation, we have

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{\rho gA(\sin\theta_1 + \sin\theta_2)}{\rho A(L_1 + L_2)}} = \frac{1}{2\pi}\sqrt{\frac{g(\sin\theta_1 + \sin\theta_2)}{L_1 + L_2}}$$ **Answer: (a)**

where mass of liquid in the tube $m = \rho V = \rho A(L_1 + L_2)$

MC10

For simple harmonic motion, $x = A\cos\omega t$ and $v = -\omega A\sin\omega t$. Substituting into the expression of the total energy,

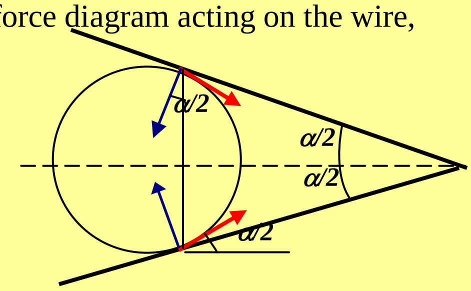
$$E = \frac{1}{2}av^2 + \frac{1}{2}bx^2 = \frac{1}{2}bA^2 + \frac{1}{2}(a\omega^2 - b)A\sin^2\omega t.$$

For energy to conserve, we have $\omega^2 = \dfrac{b}{a}$ and $f = \dfrac{\omega}{2\pi} = \dfrac{1}{2\pi}\sqrt{\dfrac{b}{a}}$. **Answer: (b)**

MC11

Denote the friction force and normal force acting on each blade as F_f and N, respectively. At the critical angle the static friction force should reach its maximal value which is $F = \mu N$.

The condition for the wire not to move is when all components of the forces adding up equal to zero. As shown in the force diagram acting on the wire,



the vertical components cancel each other by symmetry, and the horizontal components cancel if $2F_f \cos\frac{\alpha}{2} = 2N\frac{\sin\alpha}{2}$.

Thus, from the condition of the critical angle we obtain: $\mu = \tan\frac{\alpha}{2}$. **Answer: (d)**

Note that the angle in the question is twice the angle here.

MC12

Let M be the mass of the box.
If the box is not sliding, we have $F < \mu Mg$ …(1)
If the box tips over, it will happen at the lower right hand corner, so it is better to measure torques about this point. For tipping, clockwise moment must exceed counterclockwise moment:

i.e. $FH > \frac{MgL}{2}$ …. (2)

Combining (1) and (2), we have (tipping before sliding)

$$\frac{MgL}{2H} < F < \mu Mg$$

$$\Rightarrow \mu > \frac{L}{2H}$$

Thus, the critical condition in this case ($\mu > \mu_o$) is $\mu_o = \frac{L}{2H}$ **Answer: (a)**

MC13

Let v be the velocity of the particle of mass m. Kinetic energy of the compound pendulum:

$$K = \frac{1}{2}mv^2 + \frac{1}{2}(2m)\left(\frac{v}{2}\right)^2 = \frac{3}{4}mv^2$$

Potential energy of the compound pendulum:

$$U = -mgL\cos\theta - 2mg\frac{L}{2}\cos\theta = -2mgL\cos\theta$$

For small oscillations, $U \approx -2mgL\left(1-\frac{\theta^2}{2}\right) = \text{cosntant} + mgL\theta^2$

For small oscillations, we can approximate the linear displacement by $x = L\theta$.

Hence $U = \frac{mg}{L}x^2$

Using the result of MC12, we can substitute $a = \frac{3}{2}m$, $b = \frac{2mg}{L}$ and

$$f = \frac{1}{2\pi}\sqrt{\frac{2mg/L}{3m/2}} = \frac{1}{2\pi}\sqrt{\frac{4g}{3L}}$$. **Answer: (b)**

MC14

Using Newton's second law, we find that the acceleration of the upward displacement is $-g(\sin\alpha + \mu\cos\alpha)$. Hence $t_1 = \frac{v_1}{g(\sin\alpha + \mu\cos\alpha)}$ and $L = \frac{v_1^2}{2g(\sin\alpha + \mu\cos\alpha)}$.

The acceleration of the downward discplacement is $g(\sin\alpha-\mu\cos\alpha)$ (in the downward direction). Hence the time is given by $L=\frac{1}{2}g(\sin\alpha-\mu\cos\alpha)t_2^2$. Hence

$$t_2^2=\frac{2L}{g(\sin\alpha-\mu\cos\alpha)}=\frac{v_1^2}{g^2(\sin\alpha-\mu\cos\alpha)(\sin\alpha+\mu\cos\alpha)}=t_1^2\frac{\sin\alpha+\mu\cos\alpha}{\sin\alpha-\mu\cos\alpha}$$

$$\frac{t_2}{t_1}=\sqrt{\frac{\sin\alpha+\mu\cos\alpha}{\sin\alpha-\mu\cos\alpha}}$$ **Answer: (b)**

MC15

Sol. $mv_0=(m+M)v,\ v=\frac{m}{m+M}v_0$=2m/s,

$$\Delta E_k=\frac{1}{2}mv_0{}^2-\frac{1}{2}(m+M)v^2=\frac{1}{2}\times1\times10^2-\frac{1}{2}\times(1+4)\times2^2=40\text{ J}$$

Answer: (e)

MC16

Sol. 已知 m_1=1kg, m_2=3kg, F=6N, v_0 =2m/s, d =1m..

當兩物體速度相等時, 距離最小, 此時的速度為 v. 由動量守恆定律

$$m_2v_0=(m_1+m_2)v,\ v=\frac{m_2}{m_1+m_2}v_0=\frac{3}{1+3}\times2=1.5\text{ m/s.}$$

或 距離最小時所用時間為 T, $v_1=a_1T=v_0+a_2T,\ 6T=2-2T,\ T=0.25\text{s},\ v=1.5\text{m/s}$.

$$\Delta E_k=\frac{1}{2}m_2v_0{}^2-\frac{1}{2}(m_1+m_2)v^2=\frac{1}{2}\times3\times2^2-\frac{1}{2}(1+3)\times1.5^2=1.5\text{J}=F\Delta s,\ \Delta s=0.25\text{m}.$$

兩物體間的最小距離 = $d-\Delta s$ = 0.75 m. **Answer: (c)**

MC17

Let x be the fraction of the rod length submerged in water. Using Archimedes' principle, the buoyancy is given by $\rho_{\text{water}}AxLg$. Taking moments about the hanging end,

$$\rho_{\text{water}}AxLgL\left(1-\frac{x}{2}\right)=\rho_{\text{rod}}ALg\left(\frac{L}{2}\right)$$

$$x\left(1-\frac{x}{2}\right)=\frac{5}{9}\left(\frac{1}{2}\right)\quad\Rightarrow\quad x=\frac{1}{3}$$

The fraction of the rod length above the water is 2/3. **Answer: (d)**

MC18

Using $x=\frac{1}{2}at^2$, the time taken by the first carriage to pass the observer is the time difference between x = L and x = 0, where L is the length of a carriage. Hence $L=\frac{1}{2}at_0^2$ where t_0 = 5 s. The time taken by the tenth carriage to pass the observer is the time difference between x = 10L and x = 9L. Hence the time is t_2-t_1, where $9L=\frac{1}{2}at_1^2$ and $10L=\frac{1}{2}at_2^2$ $\Rightarrow$

$t_2-t_1=(\sqrt{10}-3)t_0=0.81\text{ s}$. **Answer: (e)**

MC19

Let T be the tension in the light string, and a the acceleration. Using Newton's second law,

$$F_1 - T = m_1 a \qquad (1)$$

$$T - F_2 = m_2 a \qquad (2)$$

(1) + (2): $F_1 - F_2 = (m_1 + m_2)a \quad \Rightarrow \quad a = \dfrac{F_1 - F_2}{m_1 + m_2}$ and $T = \dfrac{m_2 F_1 + m_1 F_2}{m_1 + m_2}$

$x = \dfrac{T}{k} = \dfrac{m_2 F_1 + m_1 F_2}{k(m_1 + m_2)}$. **Answer: (b)**

MC20

Consider the forces acting on the two piece of wood. The normal reactions are $N_1 = mg\cos\alpha$ and $N_2 = mg\cos\beta$.

Consider the vertical component of the forces acting on the triangular block.

$$F = Mg + N_1\cos\alpha + N_2\cos\beta = Mg + mg(\cos^2\alpha + \cos^2\beta)$$

Since $\alpha + \beta = 90^o$, $F = Mg + mg(\cos^2\alpha + \sin^2\alpha) = Mg + mg$. **Answer: (c)**

Open Problems

Q1* (10 points)

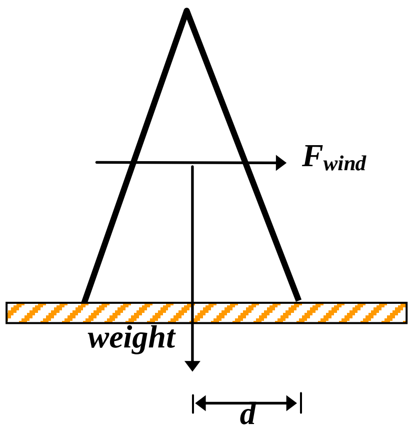
Solution:

Two forces act on the sign, namely the horizontal force, F_{wind}, from the wind and the force of gravity, mg, of the sign.

If the sign turns over, it will be pivoting at the lower right edge and there will be no longer be a force between the ground and the lower left edge. This means we need to balance the torques, about the lower right edge.

The force of gravity, acting from the center of mass, provides a stabilizing torque of

$\tau_{\text{gravity}} = F_{wind} \cdot d = (mg)(h)\tan(\theta/2) = (6\text{ kg})(9.8)(1.0)\tan15^o = 15.8\text{ Nm}$



The force from the wind acts in the center of the left board, causing a tipping torque of

$$\tau_{wind} = F_{wind}\left(\frac{h}{2}\right) \quad ..(1)$$

By using Newton's 2nd law of motion, F_{wind} can be expressed as:

$F_{wind} = \dfrac{\Delta(mv)}{\Delta t} = \dfrac{\rho A(v\Delta t)}{\Delta t} \cdot v = \rho A v^2$ ....(2) , where A is the cross-sectional area of the board.

Combining equation (1) & (2), which yields $\tau_{gravity} = \tau_{wind} = (\rho A v^2)\left(\dfrac{h}{2}\right)$

This suggests the minimum speed of the wind v_{min} is

$$v_{\min} = \sqrt{\frac{2\tau_{gravity}}{\rho A h}} = \sqrt{\frac{2(15.8)}{(1.2)(0.5)(1.0)}} = 7.26\text{ m/s}$$

Q2 (10 points)

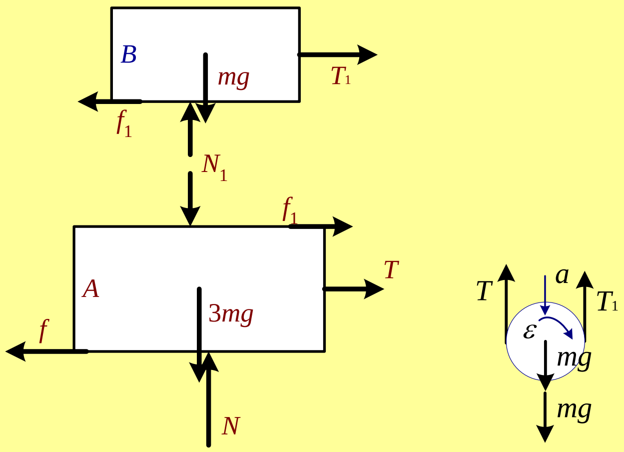
Sol.

(1) 考慮 C: $2T = mg, T = \frac{1}{2}mg$；考慮 B: $N_1 = mg,\ f_1 = T = \frac{1}{2}mg$；

考慮 A: $N = N_1 + 3mg = 4mg,\ f = f_1 + T = \frac{1}{2}mg + \frac{1}{2}mg = mg$.

系統保持平衡，則滿足: $f_1 \le \mu N_1$ 和 $f \le \mu N$ ，即

$\frac{1}{2}mg \le \mu mg$ 和 $mg \le \mu(4mg)$ ，$\mu \ge \frac{1}{2}$ 和 $\mu \ge \frac{1}{4}$ ，$\therefore\ \mu \ge \frac{1}{2}$.



設 B 加速度為 $2a$, 則 C 加速度為 a.

(2) 考慮 C: $mg - 2T = ma$； (i)

考慮 B: $N_1 = mg, T - f_1 = 2ma$, $f_1 = \mu N_1 = \mu mg$, (ii)

$\therefore \quad T - \mu mg = ma$ (iii)

由(i)和(iii), 得 $a = \frac{1-2\mu}{5}g$, $T = \frac{2+\mu}{5}mg$ (iv)

考慮 A: $N = 4mg,\ f = f_1 + T$ (v)

(ii)和(iv)代入(v), 得 $f = \mu mg + \frac{2+\mu}{5}mg = \frac{2+6\mu}{5}mg$.

C 向下做勻加速運動而 A 靜止，則滿足: $a > 0$ 和 $f \le \mu N$ ，即

$\frac{1-2\mu}{5}g > 0$ 和 $\frac{2+6\mu}{5}mg \le \mu(4mg)$ ，$\mu < \frac{1}{2}$ 和 $\mu \ge \frac{1}{7}$ ，$\therefore\ \frac{1}{2} > \mu \ge \frac{1}{7}$

Q3 (15 points)

(a) First we calculate the position of the neutral point, where the attractive forces due to the planet and the satellite are equal and opposite. Let x be the distance of the neutral point from the center of sphere B. Then

$$\frac{GMm}{x^2} = \frac{G4Mm}{(6R-x)^2} \quad \Rightarrow \quad x = 2R$$

It is sufficient to send the projectile to the neutral point. Beyond the point, the projectile can fall to the surface of sphere A under free fall. Using the conservation of energy,

$$\frac{1}{2}mv_{\min}^2 - \frac{GMm}{R} - \frac{G4Mm}{5R} = -\frac{GMm}{2R} - \frac{G4Mm}{4R} \quad \Rightarrow \quad v_{\min} = \sqrt{\frac{3GM}{5R}}$$

(b) When the projectile reaches the surface of sphere A,

$$-\frac{GMm}{2R}-\frac{G4Mm}{4R}=\frac{1}{2}mv^2-\frac{GMm}{5R}-\frac{G4Mm}{R} \quad \Rightarrow \quad v=\sqrt{\frac{27GM}{5R}}$$

(c) Escape velocity:

$$\frac{1}{2}mv_{\text{esc}}^2-\frac{GMm}{R}-\frac{G4Mm}{5R}=0 \quad \Rightarrow \quad v_{\text{esc}}=\sqrt{\frac{18GM}{5R}}$$

Q4 (15 points)

(a) Let θ be the angle between line PC and the vertical. Using the conservation of energy, the velocity of the bob at point C:

$$0=-mg\frac{L}{2}(1-\cos\theta)+\frac{1}{2}mv_C^2 \quad \Rightarrow \quad v_C=\sqrt{gL(1-\cos\theta)}$$

Using Newton's second law, $T+mg\cos\theta=m\frac{v_C^2}{L/2}=2mg(1-\cos\theta)$

$T = 0$ at point C. Therefore, $\cos\theta=\frac{2}{3} \quad \Rightarrow \quad \theta=48^\circ$

(b) At point C, the velocity is $v=\sqrt{gL(1-\cos\theta)}=\sqrt{\frac{gL}{3}}$ and makes an angle θ with the horizontal. Hence after the sting becomes loose, using point P as the origin, the coordinate of the bob is given by

$$x=\frac{L}{2}\sin\theta-v_C\cos\theta t$$

$$y=\frac{L}{2}\cos\theta+v_C\sin\theta t-\frac{1}{2}gt^2$$

Using $v_y=v_C\sin\theta-gt$, the maximum height is given by $v_y = 0 \quad \Rightarrow \quad t=\frac{v_C\sin\theta}{g}$

$$y=\frac{L}{2}\cos\theta+\frac{v_C^2\sin^2\theta}{2g}=\frac{L}{3}+\frac{gL/3}{2g}\left(\frac{5}{9}\right)=\frac{23}{54}L$$

Hence the maximum height is $23L/54$ above point P.

(c) At point E, $x = 0$. Hence

$$t=\frac{L\sin\theta}{2v_C\cos\theta}$$

$$y=\frac{L}{2}\cos\theta+\frac{\sin^2\theta}{2\cos\theta}L-\frac{3\sin^2\theta}{8\cos^2\theta}L=\frac{9}{32}L$$

Hence point E is at a distance $7L/16$ below point O.

Q5 (10 points)

(a) The system oscillates about the center of mass, which is at a distance of $L/3$ from particle A. Let a be the acceleration of A. Using Newton's second law,

$2ma=-qE\sin\theta$.

For small oscillations, $\sin\theta\approx\theta=\frac{3x}{L}$.

Hence $a=-\frac{3qE}{2mL}x$. This is the equation of a simple harmonic motion, with $\omega^2=\frac{3qE}{2mL}$.

Period of oscillations: $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{2mL}{3qE}}$

(b) The amplitude of *x*: $x_0 = \frac{L\theta_0}{3}$

The amplitude of the velocity: $v_0 = \omega x_0 = \frac{\omega L\theta_0}{3}$

Using Newton's second law, $T - qE\cos\theta = 2m\frac{v^2}{L/3}$

$T = qE\cos\theta + \frac{6mv^2}{L}$. Both terms are largest when $\theta = 0$, where $v = v_0$. Hence the maximum tension is $T_{\max} = qE + \frac{6mv_0^2}{L} = qE + qE\theta_0^2$