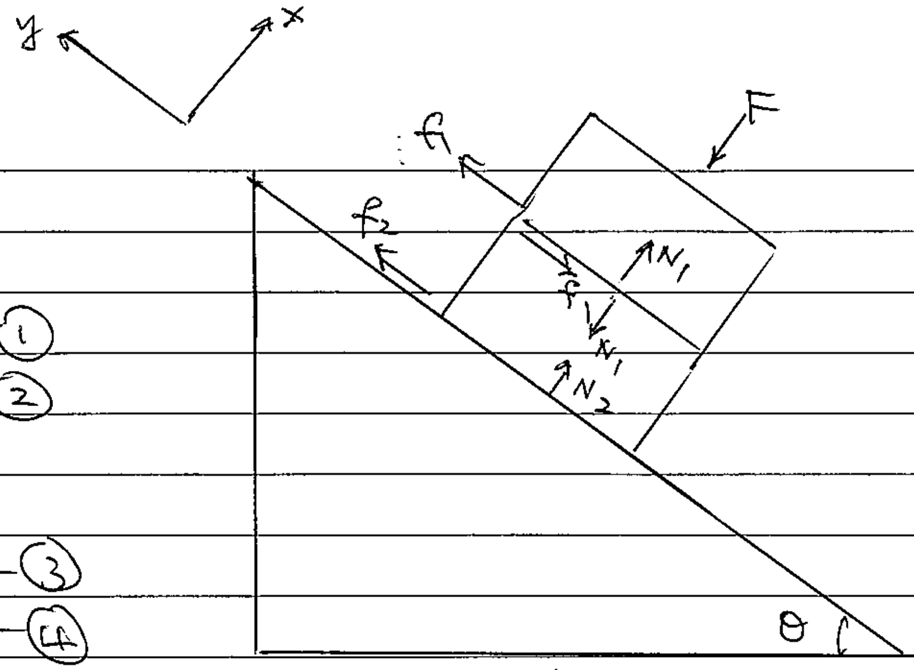
MC Key

1. ~~D~~ (Please refer to solution)
2. C
3. E
4. B
5. ~~B~~ (Cancelled)
6. A
7. D
8. D
9. C
10. A
11. B
12. E
13. D
14. B
15. D
16. A
17. A
18. C
19. E
20. D

HKPhO 2015
MC 1



y - Direction

$$mg\sin\theta = f_1 \quad \text{— (1)}$$
$$f_1 + mg\sin\theta = f_2 \quad \text{— (2)}$$

x - Direction

$$F + mg\cos\theta = N_1 \quad \text{— (3)}$$
$$N_1 + mg\cos\theta = N_2 \quad \text{— (4)}$$

$$\begin{cases} f_1 \le \mu N_1 & \text{— (5)} \\ f_2 \le \mu N_2 & \text{— (6)} \end{cases}$$

(1), (2) and (6) $\Rightarrow$ $f_2 = 2mg\sin\theta \le \mu N_2$ — (7)

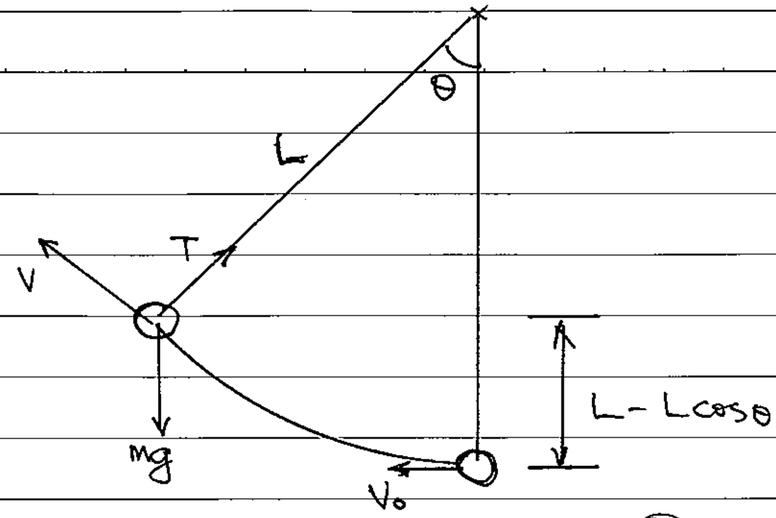
(3) and (4) $\Rightarrow$ $F + 2mg\cos\theta = N_2$ — (8)

(7) and (8) $\Rightarrow$ $\dfrac{2mg\sin\theta}{\mu} \le F + 2mg\cos\theta$

$\Rightarrow$ $F \ge \dfrac{2mg(\sin\theta - \mu\cos\theta)}{\mu}$

where F > 0

2.



$$\begin{cases} \frac{1}{2} m V_0^2 = \frac{1}{2} m v^2 + mg(L - L\cos\theta) & \text{—— (1)} \\ T - mg\cos\theta = \frac{mv^2}{L} & \text{—— (2)} \end{cases}$$

$$(1) \Rightarrow v^2 = V_0^2 - 2g(L - L\cos\theta) \quad \text{—— (3)}$$

Sub (3) into (2),

$$T - mg\cos\theta = \frac{m}{L}\left[V_0^2 - 2g(L - L\cos\theta)\right]$$

$$T = \frac{mV_0^2}{L} - 2mg(1 - \cos\theta) + mg\cos\theta$$

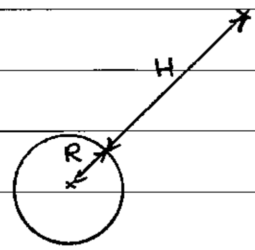
$$= \frac{mV_0^2}{L} - 2mg + 2mg\cos\theta + mg\cos\theta$$

$$= \frac{mV_0^2}{L} - 2mg + 3mg\cos\theta$$

$$= \frac{mV_0^2}{L} + mg(3\cos\theta - 2)$$

3.

$$\frac{GMm}{(R+H)^2} = m\omega^2 (R+H)$$



$$(R+H)^3 = \frac{GM}{\omega^2}$$

$$\left(\frac{R_{Earth} + H_{Earth}}{R_{moon} + H_{moon}}\right)^3 = \frac{M_{Earth}/\omega_{Earth}^2}{M_{moon}/\omega_{moon}^2}$$

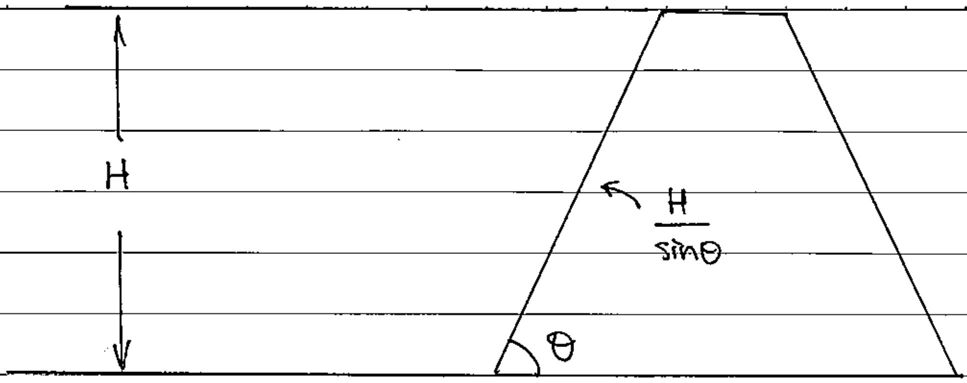
$$= \frac{M_{Earth}}{M_{moon}} \times \left(\frac{\omega_{moon}}{\omega_{Earth}}\right)^2$$

$$= (81)\left[\frac{2\pi/(27\times 24\times 3600)}{2\pi/(24\times 3600)}\right]^2$$

$$= \frac{81}{(27)^2}$$

$$= \left(\frac{1}{9}\right)^{\frac{1}{3}}$$

4.

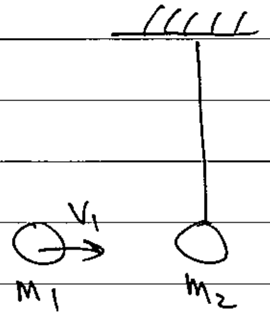


Mean Pressure $= \rho g \frac{H}{2}$

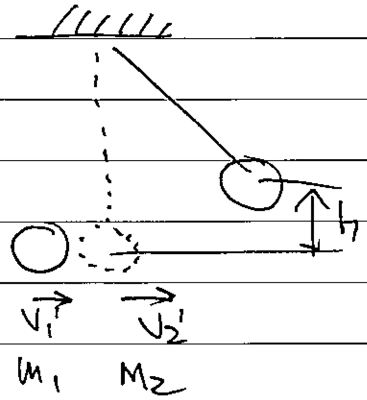
$$F = \rho g \frac{H}{2} \times \frac{H}{\sin\theta} \times W$$

$$= \frac{\rho g H^2 W}{2 \sin\theta}$$

8.



Before Collision



Head-on Elastic Collision,

$$v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_1$$

$$v_2' = \left(\frac{2m_1}{m_1 + m_2}\right) v_1$$

Energy,

$$\frac{1}{2} m_2 \left(\frac{2m_1}{m_1 + m_2}\right)^2 v_1^2 = m_2 g h$$

$$h = \frac{1}{2g}\left(\frac{2m_1}{m_1 + m_2}\right)^2 v_1^2$$

$$= \frac{2}{g}\left(\frac{m_1}{m_1 + m_2}\right)^2 v_1^2$$

9.

$$\frac{m}{t} gh + \frac{1}{2}\frac{m}{t} v^2 = P \times 0.8$$

$$\frac{\rho V}{t} gh + \frac{1}{2}\frac{\rho V}{t} v^2 = 0.8 P$$

$$\frac{(1000)(0.1)}{1}\left(9.8 \times 10 + \frac{2^2}{2}\right) = 0.8P$$

$$P = 12.5 \text{ kW}$$

11.

Work Done $= \int F\,dx$ = Area under all semi circles

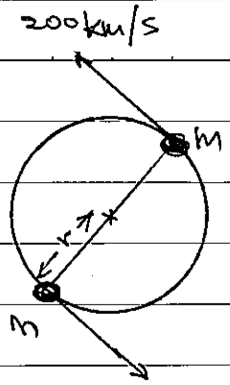
$$= \frac{1}{2}\pi r^2 + \frac{1}{2}\pi\left(\frac{r}{2}\right)^2 + \frac{1}{2}\pi\left(\frac{r}{4}\right)^2 + \cdots\cdots$$

$$= \frac{1}{2}\pi r^2\left(1 + \frac{1}{4} + \frac{1}{16} + \cdots\cdots\right)$$

$$= \frac{1}{2}\pi r^2\left(\frac{1}{1-\frac{1}{4}}\right)$$

$$= \frac{2}{3}\pi r^2$$

12.



$$\frac{Gmm}{(2r)^2} = \frac{mv^2}{r}$$

$$\Rightarrow \quad m = \frac{4v^2 r}{G} \quad \text{——— } (1)$$

$$\text{Period } T = \frac{\text{Distance travelled in one orbit}}{\text{velocity}}$$

$$= \frac{2\pi r}{v}$$

$$\Rightarrow \quad r = \frac{Tv}{2\pi}$$

$$(1) \quad \Rightarrow \quad m = \frac{4v^2}{G}\left(\frac{Tv}{2\pi}\right)$$

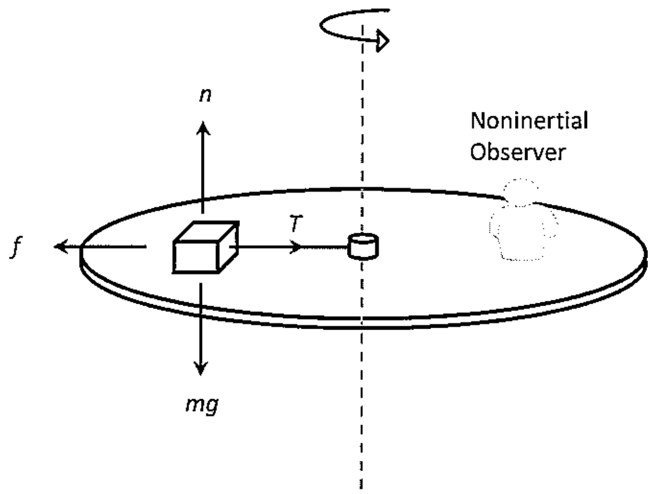
$$= \frac{2v^3 T}{\pi G}$$

$$= \frac{(2)(200\ \text{km/s})^3\,(12.2 \times 24 \times 3600)}{\pi \times 6.67 \times 10^{-11}}$$

$$= 8.05 \times 10^{31}\ \text{kg}$$

$$= 40.4 \text{ Solar Mass}$$

13. According to the noninertial observer, the block is at rest, and there is a friction force ($f = m\omega^2 r$, outward) to balance the inward force exerted by the string.



14.

$$\frac{1}{2}mv^2 - \frac{GM_Em}{R_E} = -\frac{GM_Em}{R_E+h}$$

$$\frac{(8000)^2}{2} - \frac{(6.67\times10^{-11})(5.98\times10^{24})}{6378\times10^3} = -\frac{(6.67\times10^{-11})(5.98\times10^{24})}{6378\times10^3+h}$$

$$h = 6683\ km$$

15. $R = \dfrac{V_i^2 \sin 2\theta}{g}$

$\Rightarrow \quad 2\theta = 78.5^\circ \Rightarrow \theta = 39.25^\circ$

OR

$2\theta = 101.5^\circ \Rightarrow \theta = 50.75^\circ$

16.

$$\begin{cases} T - m_1 g = m_1 a \\ m_2 g \sin\theta - T = m_2 a \end{cases}$$

$$\Rightarrow \quad a = \frac{m_2 g \sin\theta - m_1 g}{m_1 + m_2}$$

Down the slope $\Rightarrow$ $a > 0$

$$\therefore \quad m_2 g \sin\theta - m_1 g > 0$$

$$\sin\theta > \frac{m_1}{m_2}$$

17.

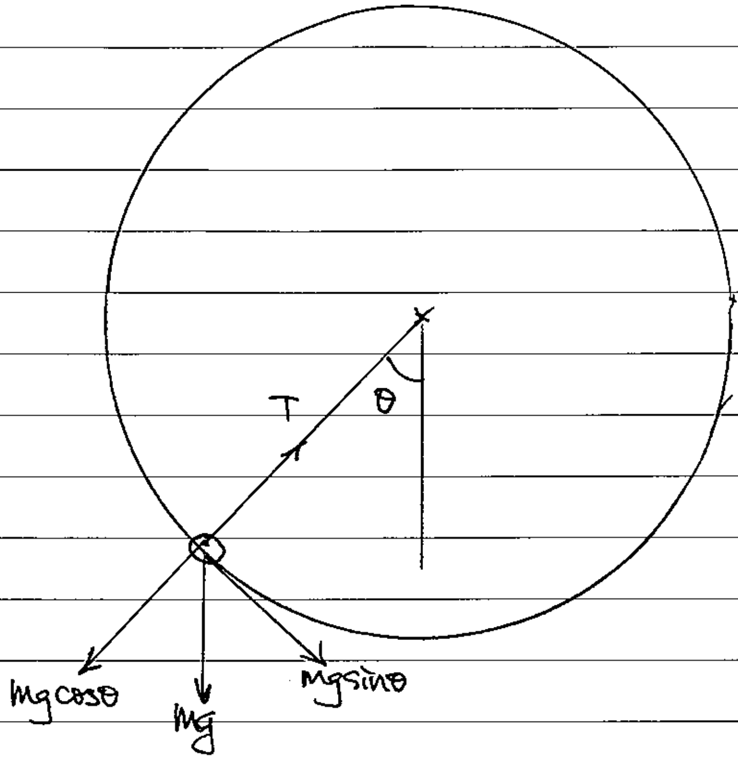
$$T - mg\cos\theta = \frac{mv^2}{R}$$

$T = 0$,

$$\Rightarrow \quad -g\cos\theta = \frac{v^2}{R}$$

At the top, $\theta = \pi$,

$$\Rightarrow \quad v^2 = gR$$



19.

$$y(t) = A\cos(-\omega t)$$

$$= A\cos\left[\frac{\pi}{2} - \left(\omega t + \frac{\pi}{2}\right)\right]$$

$$= A\sin\left(\omega t + \frac{\pi}{2}\right)$$

$$\omega = \sqrt{\frac{k}{M}} \quad \Rightarrow \quad k = \omega^2 M$$

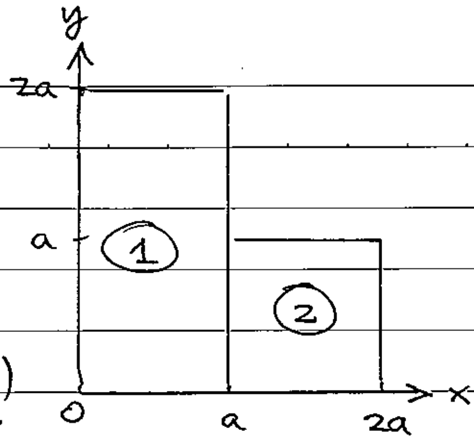
$$\text{Total Energy} = \frac{1}{2} k A^2$$

$$= \frac{1}{2}\omega^2 m A^2$$

$$= \frac{1}{2}(0.5)^2(0.2)(0.1)^2$$

$$= 0.25 \text{ mJ}$$

20.



$$\vec{r}_G = \frac{\vec{r}_1 m_1 + \vec{r}_2 m_2}{m_1 + m_2}$$

$$= \frac{\left(\frac{a}{2}\hat{i} + a\hat{j}\right)(2a^2\rho) + \left(\frac{3}{2}a\hat{i} + \frac{a}{2}\hat{j}\right)(\rho a^2)}{2\rho a^2 + \rho a^2}$$

$$= \frac{2}{3}\left(\frac{a}{2}\hat{i} + a\hat{j}\right) + \frac{1}{3}\left(\frac{3}{2}a\hat{i} + \frac{a}{2}\hat{j}\right)$$

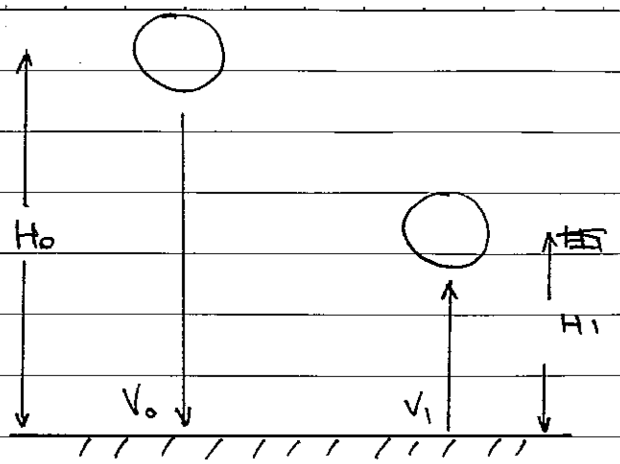
$$= \left(\frac{a}{3} + \frac{a}{2}\right)\hat{i} + \left(\frac{2a}{3} + \frac{a}{6}\right)\hat{j}$$

$$= \frac{5a}{6}\hat{i} + \frac{5a}{6}\hat{j}$$

$$= \frac{5a}{6}(\hat{i} + \hat{j})$$

Long Questions

1.



$$S = \frac{1}{2} g t^2$$

$$V_0 = g\sqrt{\frac{2H_0}{g}}$$

$$V_1 = g\sqrt{\frac{2H_1}{g}}$$

(a) $$C_R = \frac{g\sqrt{\frac{2H_1}{g}}}{g\sqrt{\frac{2H_0}{g}}} = \sqrt{\frac{H_1}{H_0}}$$

(b) $$H_{total} = H_0 + \underbrace{2H_1 + 2H_2 + \cdots\cdots}_{\text{include round trip distance}}$$

$$H_1 = C_R^2 H_0$$

$$H_2 = C_R^2 H_1 = C_R^2 C_R^2 H_0 = C_R^4 H_0$$

$$H_3 = C_R^6 H_0$$

$$H_{total} = H_0 + 2C_R^2 H_0 + 2C_R^4 H_0 + 2C_R^6 H_0 + \cdots\cdots$$

$$= H_0\left[1 + 2C_R^2 + 2C_R^4 + 2C_R^6 + \cdots\cdots\right]$$

$$= H_0\left[2\left(1 + C_R^2 + C_R^4 + C_R^6 + \cdots\right) - 1\right]$$

$$= H_0\left[2\left(\frac{1}{1-C_R^2}\right) - 1\right]$$

(c) $t_{total} = t_0 + t_1 + t_2 + \cdots\cdots$

$$t_0 = \left(\frac{2H_0}{g}\right)^{\frac{1}{2}}$$

$$t_1 = 2\left(\frac{2H_1}{g}\right)^{\frac{1}{2}}$$

$$= 2\left(\frac{H_1}{H_0}\right)^{\frac{1}{2}}\left(\frac{2H_0}{g}\right)^{\frac{1}{2}}$$

$$= 2\,C_R\left(\frac{2H_0}{g}\right)^{\frac{1}{2}}$$

$$t_2 = 2\left(\frac{2H_2}{g}\right)^{\frac{1}{2}}$$

$$= 2\left(\frac{H_2}{H_1}\right)^{\frac{1}{2}}\left(\frac{H_1}{H_0}\right)^{\frac{1}{2}}\left(\frac{2H_0}{g}\right)^{\frac{1}{2}}$$

$$= 2\,C_R^2\left(\frac{2H_0}{g}\right)^{\frac{1}{2}}$$

where $t_1, t_2, \cdots$ include round trip time

$$t_{total} = \left(\frac{2H_0}{g}\right)^{\frac{1}{2}}\left[2\left(1 + C_R + C_R^2 + C_R^3 + \cdots\right) - 1\right]$$

$$= \left(\frac{2H_0}{g}\right)^{\frac{1}{2}}\left[2\left(\frac{1}{1 - C_R}\right) - 1\right]$$

2.

(a) x-component: $T_2 \sin\theta_2 - T_1 \sin\theta_1 = 0$

$$\Rightarrow \quad T_2 = T_1 \frac{\sin\theta_1}{\sin\theta_2} \quad \text{——— (1)}$$

y-component: $T_1 \cos\theta_1 + T_2 \cos\theta_2 - 3\,mg = 0$ ——— (2)

About the position of the load:

$$-x T_1 \cos\theta_1 - \left(\frac{L}{2} - x\right) mg + (L - x) T_2 \cos\theta_2 = 0 \quad \text{——— (3)}$$

(1) and (2)

$$\Rightarrow \quad T_1 \cos\theta_1 + T_1 \frac{\sin\theta_1}{\sin\theta_2} \cos\theta_2 - 3\,mg = 0$$

$$T_1 \left(\cos\theta_1 + \frac{\sin\theta_1}{\sin\theta_2} \cos\theta_2 \right) = 3\,mg$$

$$T_1 \left(\frac{\sin\theta_2 \cos\theta_1 + \sin\theta_1 \cos\theta_2}{\sin\theta_2} \right) = 3\,mg$$

$$T_1 \frac{\sin(\theta_1 + \theta_2)}{\sin\theta_2} = 3\,mg$$

$$T_1 = \frac{3\,mg \sin\theta_2}{\sin(\theta_1 + \theta_2)}$$

$$T_2 = T_1 \frac{\sin\theta_1}{\sin\theta_2}$$

$$= \frac{3\,mg \sin\theta_2}{\sin(\theta_1 + \theta_2)} \cdot \frac{\sin\theta_1}{\sin\theta_2}$$

$$= \frac{3\,mg \sin\theta_1}{\sin(\theta_1 + \theta_2)}$$

(b) ③ $\Rightarrow$ $\left(-T_1\cos\theta_1 + mg - T_2\cos\theta_2\right)x = \frac{L}{2}mg - T_2 L\cos\theta_2$

$$\left[-3mg\frac{\sin\theta_2}{\sin(\theta_1+\theta_2)}\cos\theta_1 + mg - 3mg\frac{\sin\theta_1}{\sin(\theta_1+\theta_2)}\cos\theta_2\right]x$$
$$= \frac{L}{2}mg - 3mg\frac{\sin\theta_1}{\sin(\theta_1+\theta_2)}L\cos\theta_2$$

$$\left[\frac{-3mg\sin\theta_2\cos\theta_1 + mg\sin(\theta_1+\theta_2) - 3mg\sin\theta_1\cos\theta_2}{\sin(\theta_1+\theta_2)}\right]x$$
$$= \frac{\frac{L}{2}mg\sin(\theta_1+\theta_2) - 3mg\sin\theta_1 L\cos\theta_2}{\sin(\theta_1+\theta_2)}$$

$$\left[-3\sin\theta_2\cos\theta_1 + \sin(\theta_1+\theta_2) - 3\sin\theta_1\cos\theta_2\right]x$$
$$= \frac{L}{2}\sin(\theta_1+\theta_2) - 3L\sin\theta_1\cos\theta_2$$

$$\left[\sin(\theta_1+\theta_2) - 3\sin(\theta_1+\theta_2)\right]x = \frac{L}{2}\sin(\theta_1+\theta_2) - 3L\sin\theta_1\cos\theta_2$$

$$x = \frac{\frac{L}{2}\sin(\theta_1+\theta_2) - 3L\sin\theta_1\cos\theta_2}{-2\sin(\theta_1+\theta_2)}$$
$$= \frac{3L}{2}\frac{\sin\theta_1\cos\theta_2}{\sin(\theta_1+\theta_2)} - \frac{L}{4}$$

(c) $\theta_1 + \theta_2 = 90^\circ$

$$\Rightarrow \quad x = \frac{3L}{2}\sin\theta_1\cos\theta_2 - \frac{L}{4}$$
$$= \frac{3L}{2}\left[\frac{1}{2}\left(\sin(\theta_1-\theta_2) + \sin(\theta_1+\theta_2)\right)\right] - \frac{L}{4}$$
$$= \frac{3L}{4}\sin(\theta_1-\theta_2) + \frac{3L}{4} - \frac{L}{4}$$
$$= \frac{3L}{4}\sin(\theta_1-\theta_2) + \frac{L}{2}$$

(c) $x = \frac{L}{8}$

$$\Rightarrow \quad \frac{L}{8} = \frac{3L}{4} \sin(\theta_1 - \theta_2) + \frac{L}{2}$$

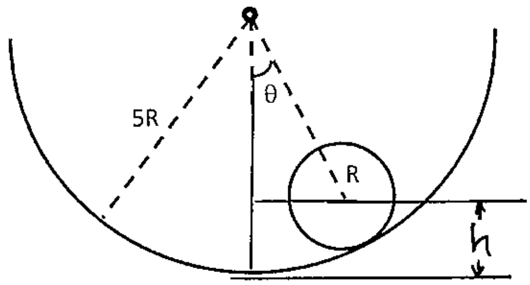
$$\sin(\theta_1 - \theta_2) = -\frac{1}{2}$$

$$\begin{cases} \theta_1 - \theta_2 = -30^\circ \\ \theta_1 + \theta_2 = 90^\circ \end{cases}$$

$$\Rightarrow \quad \theta_1 = 30^\circ$$
$$\theta_2 = 60^\circ$$

3.

(a) $KE = \frac{56}{5} mR^2 \left(\frac{\Delta\theta}{\Delta t}\right)^2$



For a displacement θ,

$h = 4R(1-\cos\theta)$

$PE = mgh = 4mgR(1-\cos\theta)$

(b) For small angle, $(1-\cos\theta) \approx \frac{\theta^2}{2}$

$\Rightarrow \quad PE \approx 4mgR\theta^2$

$$\text{Total Energy} = \text{Constant} = E = \frac{56}{5} mR^2 \left(\frac{\Delta\theta}{\Delta t}\right)^2 + 4mgR\theta^2$$

$$= \frac{1}{2}\left(\frac{112}{5} mR^2\right)\left(\frac{\Delta\theta}{\Delta t}\right)^2 + \frac{1}{2}(4mgR)\,\theta^2$$

(c) $M_{eff} = \frac{112}{5} mR^2$

$k_{eff} = 4mgR$

$$\omega = \sqrt{\frac{k_{eff}}{M_{eff}}} = \sqrt{\frac{4mgR}{112/5\, mR^2}} = \sqrt{\frac{5}{28}\frac{g}{R}}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{28R}{5g}}$$

Alternative Approach

$$\frac{dE}{dt} = \left(\frac{112}{5} mR^2\right)\ddot{\theta} + 4mgR\dot{\theta} = 0$$

$$\Rightarrow \quad \ddot{\theta} = -\frac{5}{28}\frac{g}{R}\dot{\theta}$$

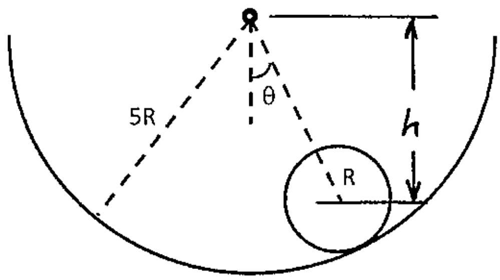
$$\omega = \sqrt{\frac{5}{28}\frac{g}{R}}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{28}{5}\frac{R}{g}}$$

3. Alternative Approach for (3a) and (3b)

(a) For students who use the circle center as the origin:

$$h = 4R\cos\theta$$



$$PE = mgh$$
$$= -4mgR\cos\theta$$

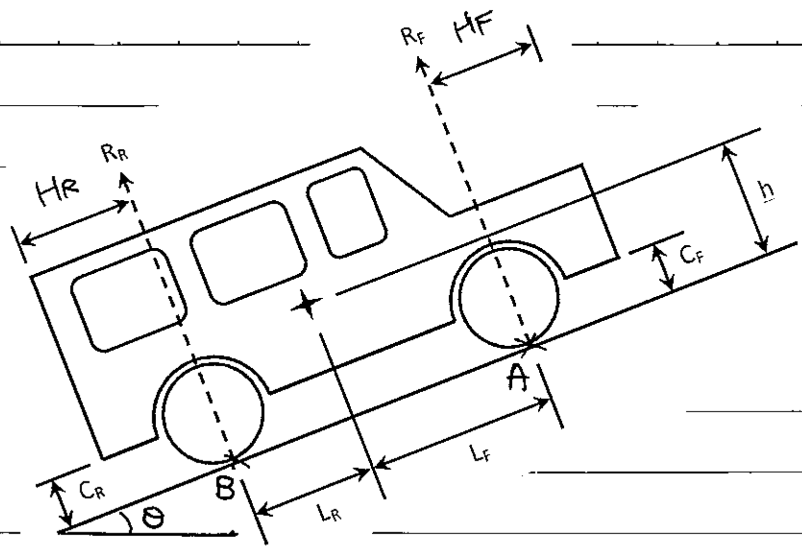
(b) For small angle, $\cos\theta \approx 1 - \frac{\theta^2}{2}$

$$PE + KE = \text{constant}$$

$$E = \frac{56}{5} mR^2 \left(\frac{\Delta\theta}{\Delta t}\right)^2 - 4mgR\cos\theta$$
$$= \frac{56}{5} mR^2 \left(\frac{\Delta\theta}{\Delta t}\right)^2 - 4mgR\left(1 - \frac{\theta^2}{2}\right)$$
$$= \frac{56}{5} mR^2 \left(\frac{\Delta\theta}{\Delta t}\right)^2 - 4mgR + 2mgR\theta^2$$

$$\Rightarrow E + 4mgR = \text{constant} = \frac{56}{5} mR^2 \left(\frac{\Delta\theta}{\Delta t}\right)^2 + 2mgR\theta^2$$
$$= \frac{1}{2}\left(\frac{112}{5} mR^2\right)\left(\frac{\Delta\theta}{\Delta t}\right)^2 + \frac{1}{2}(4mgR)\theta^2$$

4.



(i) <u>Rear Wheel Drive</u>

Moment about A :

$$mg\cos\theta\, L_F + mg\sin\theta\, h - R_R L = 0 \quad \text{—— (1)}$$

Friction :

$$f = mg\sin\theta$$

$$\text{Adhesion} \Rightarrow f_{max} = \mu R_R = mg\sin\theta$$

$$R_R = \frac{mg\sin\theta}{\mu} \quad \text{—— (2)}$$

$$\text{(1) and (2)} \Rightarrow mg\cos\theta\, L_F + mg\sin\theta\, h - \frac{mg\sin\theta}{\mu} L = 0$$

$$L_F + h\tan\theta - L\frac{\tan\theta}{\mu} = 0$$

$$\tan\theta\left(h - \frac{L}{\mu}\right) = -L_F$$

$$\tan\theta = \frac{\mu L_F}{L - \mu h}$$

(ii) <u>Front Wheel Drive</u>

Moment about B :

$$- mg\cos\theta\, L_R + mg\sin\theta\, h + R_F L = 0 \quad \text{——} \quad ③$$

Friction :

$$f = mg\sin\theta$$

$$\text{Adhesion} \Rightarrow \quad f_{max} = \mu R_F = mg\sin\theta$$

$$R_F = \frac{mg\sin\theta}{\mu} \quad \text{——} \quad ④$$

③ and ④

$$\Rightarrow \quad - mg\cos\theta\, L_R + mg\sin\theta\, h + \frac{mg\sin\theta}{\mu} L = 0$$

$$- L_R + h\tan\theta + L\frac{\tan\theta}{\mu} = 0$$

$$\tan\theta \left(h + \frac{L}{\mu} \right) = L_R$$

$$\tan\theta = \frac{\mu L_R}{L + \mu h}$$

(iii) <u>Four Wheel Drive</u>

Friction : $f = mg\sin\theta$ —— (5)

$f_{max} = \mu mg\cos\theta$ —— (6)

(5) and (6) $\Rightarrow$ $mg\sin\theta = \mu mg\cos\theta$

$$\tan\theta = \mu$$

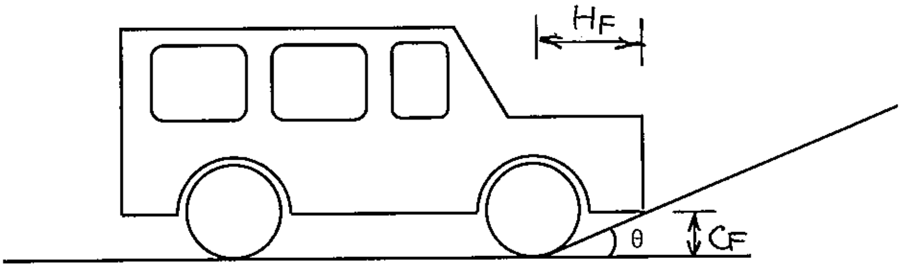
(iv) <u>Overturn</u>

Overturn $\Rightarrow$ $R_F = 0$

(3) $\Rightarrow$ $-mg\cos\theta\, L_R + mg\sin\theta\, h = 0$

$$\tan\theta = \frac{L_R}{h}$$

(V) <u>Clearance</u>

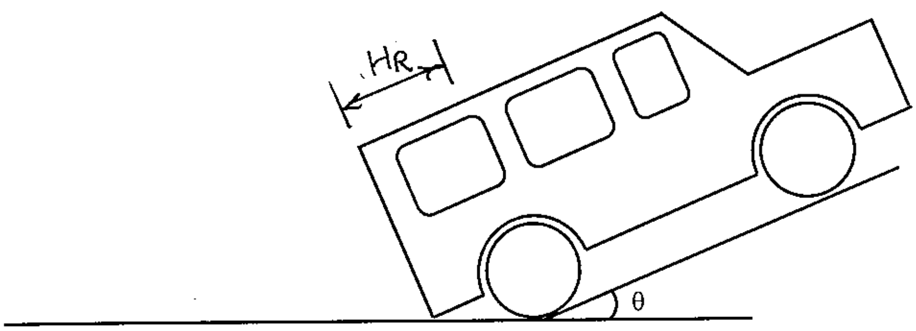


Front :

$$\tan\theta = \frac{C_F}{H_F}$$

Rear :

$$\tan\theta = \frac{C_R}{H_R}$$



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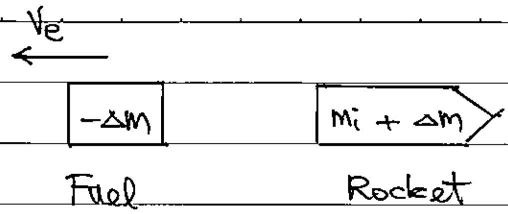

Remack: * At time $t + \Delta t$, rocket mass changes from m_i to $(m_i + \Delta m)$

* Δm is negative for decrease in rocket mass.

(a) Momentum : $-\Delta m V_e = (m_i + \Delta m) \Delta V$

$$-\Delta m V_e = m_i \Delta V + \cancel{\Delta m \Delta V}$$

$$\Delta V = -V_e \frac{\Delta m}{m_i}$$

(b) Acceleration $a = \frac{\Delta V}{\Delta t} = -V_e \frac{\Delta m}{m_i} \left(\frac{1}{\Delta t} \right)$

$$F = m_i a = -V_e \frac{\Delta m}{\Delta t}$$

$$\frac{\Delta m}{\Delta t} = \frac{-F}{V_e}$$

$$= \frac{-3.5 \times 10^7}{2500}$$

$$= -14000 \text{ kg/s}$$